Targeted Advertising and Consumer Inference

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Abstract

The mere fact that consumers are targeted by advertisements can affect their inference about the expected utility of a product. We build a micro-model where multiple firms compete through targeted advertising. Consumers make inferences from targeted advertising about their potential match values for the product category, as well as the advertising firm's unobserved quality. We show that in equilibrium, upon being targeted by a firm, consumers make optimistic inferences about the product category and the firm's quality. With such improved beliefs, a targeted consumer is more likely to engage in a costly search throughout the category. We find that the increase in consumer search creates an advertising spillover beyond the level of the mere awareness effects of advertising and that firms' equilibrium level of targeted advertising can be nonmonotonic in targeting accuracy. Additionally, we show that sometimes, it can be optimal for firms to relinquish customer data and instead engage in non-targeted advertising. The results provide insights into the trade-offs between advertising reach and targeting accuracy.

Keywords: targeted advertising, targeting accuracy, consumer inference, consumer search, reach and precision, value of information, prominence, free-riding

1 Introduction

In the past few decades, online targeting technology has grown increasingly important in marketing practice. Using increasingly granular customer data, firms can identify customers who are more likely to need their products or services and benefit from the product category (Davenport et al., 2001; Braun and Moe, 2013; Summers et al., 2016). For example, advertisers on Facebook can use customers' demographic information (e.g., age, gender, and location), social activities on the platform (e.g., wall posts, clicked ads, "likes", and "sharing"), and social networks (e.g., who are their friends and what they do and like)[1] to *target* the advertisers' desired customer group. Using big data combined with a prediction algorithm that can turn data into insights about customers' potential category match by pooling a variety of information about other consumers, firms can identify those target customers and reach out to them even before consumers themselves become aware of their needs and wants (Agrawal et al., 2018; Lu and Shin, 2018).[2] As a result, consumers with a strong preference can be exposed to targeted advertisements about the products they have hitherto been unaware of. In contrast, those with preferences that do not match the product's appeal will be filtered out.

Research has shown that digital targeting meaningfully improves consumers' responses to advertisements (John et al., 2018). When customer data is used,[3] targeted advertising becomes more effective in increasing both click-through rates and conversion rates (e.g., Joshi et al., 2011; Lambrecht and Tucker, 2013; Summers et al., 2016; Yan et al., 2009), and thus, numerous firms invest in online targeted advertising.[4] Such targeted advertising is particularly crucial in uncommon or nascent product categories where consumers' default engagement levels are generally low. In these circumstances, firms can use targeted advertising to induce consumer engagement and preempt demand by identifying the prospects who are more likely to exhibit interest in the product category.

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[1] "How Facebook ads target you" at https://www.cnbc.com/2018/04/14/how-facebook-ads-target-you.html.

[2] Even if a consumer may have superior information about her preferences, she may not necessarily know about all the products available that fit her potential needs and wants. Therefore, firms can predict a product match with consumers even before the latter recognize it. For example, Amazon Family, a service for the new parent segment of those who are unaware of all the products available for newborn babies, provides automatic suggestions about what new parents might buy, based on the purchase history of not only such new parents but also all other new parents.

[3] Given Facebook's recent scandal involving Cambridge Analytica, privacy issues raise significant concerns for both marketers and consumers. As a result, many firms such as Facebook and Google try to avoid using sensitive information such as race and health conditions. Privacy issues and their effects on information sharing (in particular, third-party data sharing; cf. Goldfarb and Tucker, 2011b, Goldfarb and Tucker, 2012, Tucker, 2012) are important topics. However, they are not the focus of the current study, and we leave them for future research.

[4] In 2017, Google garnered $35 billion in the US market, which represented an increase of 18.9% over the previous year, and Facebook captured $17.37 billion (https://www.emarketer.com/Article/Google-Facebook-Tighten-Grip-on-US-Digital-Ad-Market/1016494).

However, targeting such potential customers in earlier stages of their decision-making process can be risky. Sometimes, less than 50% of qualified leads initiated by a brand's targeted advertising enter the final purchase stage (Court et al. 2009 *McKinsey Quarterly*). In this early phase of consumers' decision journeys,[5] firms need to convince and encourage them to deliberate their potential needs, thus increasing product acceptance (Lu and Shin, 2018). The efforts to identify and attract prospective customers who have yet to understand the product's uses, benefits and relevance to their needs can be substantial. Moreover, these efforts would be wasted if consumers eventually buy from another firm. Competitors with products in the same category could then benefit from the firm's advertising efforts to enhance customer awareness and interest in the product category. This positive spillover effect of one firm's advertising on the competitor's brand is more than a theoretical possibility. Such a spillover effect in advertising has been well documented in several empirical studies that show that one firm's targeted advertising can prime consumers to think about the product category, benefiting competing firms (Anderson and Simester, 2013; Lewis and Nguyen, 2015; Sahni, 2016; Shapiro, 2018).

Consider the following incident, which we use as our running example for the paper's model and assumptions. One of the authors recently came across an advertisement on Facebook, featuring a new scanning app for iPhones. He clicked the advertisement and downloaded the free version of the app. Although he did not like this particular app (notably, he did not even know that such a product existed and, after a few trials, could not appreciate the value of its mobile scanning function over a simple camera), he was aware that Facebook ads are often highly relevant. Thus, instead of ignoring the mobile scanning function entirely, he further searched for other scanning apps in Google. He then realized that it could be useful to scan documents instantaneously and export them as multipage PDF files. As a result, he purchased a different scanning app with such useful functions. Clearly, targeted advertising by one seller has motivated this author's interest in such a product category. Without this targeted advertisement, he would not have engaged in any search in the category. However, the firm's targeted advertising benefited its competitor, which free-rode on the firm's costly advertising efforts.

In this incident, targeted advertising was key to engaging consumers and creating the category demand. Facebook's targeting ability based on customer information helps firms reach prospects

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[5]The customer journey (Court et al. (2009), Lemon and Verhoef (2016), and Richardson (2010)) is an idea that conceptualizes customer experience as a "journey" with a firm over time during the purchase cycle across multiple touchpoints. The literature on customer journey emphasizes the purchase funnel, e.g., in the Awareness-Interest-Desire-Action (AIDA) model. In particular, Shin (2005) conceptualizes the costs associated with the early stage of a purchase funnel as selling costs and its importance in the sales process.

with better fit, considering product features and benefits. Facebook might have first filtered people to identify specific consumers who are in education (students or academics) or business for whom document scanning could be useful in reducing the amount of paperwork and the need for filing cabinets and improving data security and protection. In the example above, the mere fact that the advertisement was targeted made us more interested in the product category and eventually led to a product purchase. For instance, if it were an advertisement in a newspaper (which is not targeted toward a specific consumer), it would not have necessarily engaged consumers in such endeavors. The distinct response of consumers to such targeted advertising implies that they (consciously or unconsciously) acknowledge its *relevance*, and make inferences based on the fact that they are targeted. We focus on a mechanism that triggers this additional effect of targeting through consumers' inferences beyond the simple advertising effect of increasing awareness. An essential determinant of consumers' inference from the fact that they are targeted is *targeting accuracy*. We build a game-theoretic model to formally study how targeting accuracy affects consumers' inference processes and subsequent search behaviors when there are multiple firms in the market.

We begin by providing a micro-model of the consumer inference process when consumers encounter targeted advertising. In particular, consumers observe one signal of such advertising and update their beliefs about *two unknowns*–their own match type for the product category and the advertisers' quality types. We pay special attention to the effects of targeting accuracy on the consumer's belief-updating processes along these two dimensions due to targeted advertising.

Armed with this understanding, we investigate the firm's optimal targeted advertising strategy and how it affects consumers' searching and purchasing decisions. Several factors affect a firm's decision to invest in targeted advertising based on the level of accuracy. On the one hand, as a firm advertises more, more consumers consider it because of its prominence in the market (Armstrong et al., 2009), which helps preempt demand for its competition. If the accuracy of targeting improves, the value of prominence improves as more consumers will be satisfied with their initial search, and make purchases immediately. Additionally, accuracy can reduce the costs of targeting by avoiding consumers unlikely to be interested in the product category (Goldfarb, 2014), thus eliminating "wasted" advertising efforts. These two benefits are the direct effects of improved accuracy.

On the other hand, there is another indirect effect of improved accuracy, which may prove harmful to firms: namely, free-riding of competitors due to increased consumer search (Shin, 2007). As target-

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ing accuracy increases, beliefs about product categories become more optimistic, and consumers are encouraged to search for other alternatives beyond the prominent firm. Thus, enhancing consumers' belief about the product's value via targeted advertising may inadvertently benefit the firm's competitors. The effects of this phenomenon can be substantial. Firms thus face the prospect of having their initial targeted advertising investments wasted, thereby significantly reducing their incentives to invest in advertising. The more accurate the targeting technology is, the greater this indirect effect is. For these reasons, the effects of improved targeting capability on the firms' incentives to engage in targeted advertising are ambiguous.

Even without targeting, the trade-off between prominence and free-riding effects still exists when firms compete in advertising because the advertising of one firm can activate consumers' awareness of other firms. Targeting essentially amplifies this trade-off by affecting consumers' beliefs about the product category. This raises interesting questions of whether and which firms should lead efforts in targeted advertising. In equilibrium, we show that a firm of higher quality invests more aggressively in targeted advertising to consumers likely to benefit from the product category. Therefore, upon being targeted by a firm's advertising, consumers rationally make more optimistic inferences about both their match value for the product category and the quality of the firm. We also find several implications of targeting accuracy on equilibrium outcomes.

First, we find that targeting accuracy can have nonmonotonic effects on the equilibrium level of advertising. It implies that even if targeting becomes more accurate, and if firms can identify valuable consumers with higher precision, firms may find it optimal to spend less on targeted advertising. In particular, when consumers, on average, have a lower search cost, greater accuracy in targeting increases consumer search, further reducing the firm's incentive to invest in advertising in equilibrium. However, when the consumer search cost is sufficiently high, this result does not hold because firms compete and invest aggressively in targeted advertising as free-riding becomes less significant.

Second, we find that firms may spend more on targeted advertising than in the case of non-targeted advertising. One may expect firms to spend less on targeted advertising than non-targeted advertising. By definition, targeted advertising can be sent to a smaller number of potential customers who might have a good match with the product category. However, high accuracy in targeting can lead to more aggressive investments, and therefore, its equilibrium level of advertising reach can exceed that of the non-targeted advertising case. Moreover, we observe that if targeting accuracy

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is not sufficiently high, firms can be better off relinquishing customer data and instead engaging in non-targeted advertising.[6] This result provides an insight into a recent debate on how companies need to balance between advertising reach and targeting precision.[7] Our finding suggests that in a product category with a narrow market appeal (a niche product category), targeted advertising with higher precision is increasingly more profitable. However, in a product category with a broad market appeal (a mass-market product category), non-targeted advertising becomes more appealing.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model of targeted advertising under competition with exogenous pricing. We analyze the model by focusing on consumers' inference processes, characterizing consumers' optimal search decisions, and identifying the equilibrium advertising strategy in Section 4. Section 5 extends the main model by endogenizing pricing. Section 6 concludes.

2 Literature Review

This paper is closely related to the literature on targeted advertising and consumer search. First, the literature on online advertising has emphasized the importance of targeting using various customer data, such as demographic information (Joshi et al., 2011), cognitive styles (Hauser et al., 2009), browsing behaviors such as ad clicks (e.g., Agarwal et al., 2009; Chen et al., 2009), or past purchases (e.g., Rossi et al., 1996; Fader et al., 2005). Research in this area consistently finds that tailoring the message based on the target segment's characteristics or specific content of a website improves the performance of communication and consumer response (Goldfarb and Tucker, 2011a, Zhang and Katona, 2012). Our paper contributes to this literature by providing a micro-foundation for such effectiveness based on a consumer's rational inference process from the mere fact of getting targeted.[8]

There is another strand of advertising research that investigates the effects of targeting accuracy on the equilibrium outcomes. An early contribution in this area is Chen et al. (2001). The authors show that imperfect targeting can soften price competition among firms. The reason is that firms

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[6] Several papers also find that the advertiser may be better off with less information in cheap-talk communication(Gardete and Bart, 2018) or in sending quality signals to encourage consumer search (Mayzlin and Shin, 2011).

[7] https://www.wsj.com/articles/p-g-to-scale-back-targeted-facebook-ads-1470760949.

[8] A recent paper by Kuksov and Liao (2018) is close to ours in studying consumer inference. Even though their paper is not about the effect of targeting or targeting accuracy, which is the focus of our research, their model captures the consumer inference process when opinion leaders post their reviews, which could be the outcomes of either vertical quality evaluation or horizontal taste matching. They study the firm's optimal product line decision when opinion leaders' reviews impact consumers' beliefs and purchase decisions.

can misconceive price-sensitive consumers (switchers) as price-insensitive consumers (loyal customers) and, therefore, would charge higher prices than in the case of perfect targeting. Iyer et al. (2005) show that, with targeted advertising, firms can be better off than in the no-targeting case because of differentiation. For similar reasons, Bergemann and Bonatti (2011) show that the equilibrium price of advertising can decrease in targeting accuracy even though its marginal product is increasing in targeting accuracy. From the perspective of an ad platform, Levin and Milgrom (2010) argue that platforms have incentives to limit advertisers' access to detailed customer data to make them less differentiated. Zhong (2016) studies a similar issue when the platform can control the accuracy of consumer search technology. He investigates how it affects the consumer search and its implications on firms' prices and platform revenue. Rafieian and Yoganarasimhan (2017) document empirical evidence to support an ad platform's incentives to withhold information from advertisers. Like these papers, we study the effects of targeting accuracy on the firm's equilibrium advertising strategy. However, our focus is on the micro-process of consumer inference from getting targeted, and we investigate the trade-offs between targeting accuracy and advertising intensity.[9]

Our model builds on the recent developments in consumer search theory where the search is non-random and in a deliberate order (see Armstrong, 2017 for an extensive review of the ordered search literature). Armstrong et al. (2009) demonstrates that when consumers engage in costly search across firms, *prominence*, or being the first shopping destination, can be valuable because it can preempt demand. Several papers endogenize prominence by allowing firms to obtain it, for example, by charging a lower price than others (Armstrong and Zhou, 2011; Chen and He, 2011; Zhou, 2011). In our model, firms compete to obtain prominence through advertising, and consumers engage in an ordered search, which endogenously creates advertising spillover effects and firms' free-riding incentives.

Also, several papers consider those spillover effects of advertising. Anderson and Simester (2013) conduct field experiments and show that sending a competitor's catalog can increase the focal firm's sales due to the spillover effects of advertising. Lewis and Nguyen (2015) observe that display ads on Yahoo can increase searches for competitors' brands, and such spillovers can reduce firms' investments in advertising. Sahni (2016) shows that search advertising can remind consumers of other products that might compete with the advertiser in randomized experiments. Moreover, Shapiro (2018) finds positive spillovers on rivals' demand due to TV advertising in the pharmaceutical industry. Therefore, firms

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[9]A stream of research extends the domain of targeting beyond advertising and focuses on customized pricing based on customers' past purchase history (Fudenberg and Tirole, 2000; Villas-Boas, 1999; Shin and Sudhir, 2010)

can free-ride off other firms' advertising efforts. We provide a new mechanism of how such spillover effects can arise based on consumers' rational inference from targeted advertising, and analyze firms' optimal advertising strategy in the presence of such spillover effect and free-riding incentives.

A recent paper by Summers et al. (2016) is closely related to our study. Both focus on the micro-mechanism of consumer inference from merely receiving a targeted ad, which influences the extent of consumer search in the product category. Summers et al. (2016) demonstrate this phenomenon through a series of behavioral experiments. In their work, the mechanism is a psychological mechanism of social labeling. Consumers receive information about how they are perceived by others (by targeted advertising), resulting in adjustments to self-perception and behaviors consistent with the label. On the other hand, the mechanism in this paper is based on Bayesian updating by rational consumers who recognize the relevance of targeted advertising, which utilizes customer data.

3   Model

There are two firms, denoted by $j \in \{A, B\}$, in the same product category. Both firms sell a product to a unit mass of atomless consumers. A consumer ("she") may be a bad match with the product category and therefore cannot benefit from buying any product in this category. On the other hand, she may be a good match with the product category and enjoy a product in this category if the product satisfies her needs. The following utility function captures this idea:

$$u_{ij} - p_j = \phi\left(m_i \cdot v_j\right) - p, \tag{1}$$

where u_{ij} is consumer i's consumption utility from firm j's product with price p_j; we first assume that the two products' prices are exogenously fixed at $p_A = p_B = p$ (e.g., prices of most mobile apps are similar at \$0.99). In our extension, we relax this assumption and endogenize the firm's prices. Consumption utility u_{ij} is the outcome of two parts, namely, consumer-specific $m_i \in \{0, 1\}$ and product-specific $v_j \in \{0, 1\}$, intended to capture the fact that a consumer can receive consumption utility only if the *consumer* has a good match with the product category, and the *product* satisfies the consumer's needs. In this case, she receives a positive value $\phi > 0$, which we normalize to 1 without loss of generality. Therefore, $u_{ij} \in \{0, 1\}$.

First, $m_i \in \{0, 1\}$ is consumer i's *category* match for this particular product category, where $m_i = 1$

if consumer i is of a *good* match-type for the product category, whereas $m_i = 0$ if the match-type is *bad*. This category match-type follows a common distribution such that $\Pr(m_i = 1) = \mu_0 \in [0, 1]$, but the realization is unknown to the consumer. Second, $v_j \in \{0, 1\}$, firm j's *product* match to a consumer, takes the value $v_j = 1$ if it satisfies the customer's specific needs, and otherwise, $v_j = 0$. The realization of v_j depends on quality type of firm j, denoted by q_j, which is drawn independently from a standard uniform distribution $U[0, 1]$. More precisely, the product match is $v_j = 1$ with probability q_j, and $v_j = 0$ with probability $1 - q_j$. Therefore, a higher-quality firm's product is able to meet consumers' needs with a greater probability.

Once consumer i visits firm j's website, she learns $u_{ij} \in \{0, 1\}$, i.e., whether or not she likes the firm's product. If $u_{ij} = 1$, she knows this product category is a good match ($m_i = 1$) *and* the product is sufficiently good that it satisfies her specific need ($v_j = 1$). However, if $u_{ij} = 0$, she is unable to identify the source of displeasure. She does not separately observe the exact realization of the consumer's own match-type (m_i) and product value (v_j). This is a critical assumption in our model, which implies that if a consumer has a bad experience with a product, she makes inferences about her own match-type with the product category and the firm's unobserved quality type. Based on this two-dimensional inference from being targeted, the consumer will make subsequent decisions.[10]

Information and Targeting Technology

Firms have access to customer data, which provide a noisy signal $s_i \in \{g, b\}$ for m_i, or the consumer i's true match type for the product category. We assume that both firms have access to the same data from a platform, such as Facebook, or a web publisher, such as the New York Times. Therefore, both firms receive a *common* signal about each consumer.[11] Based on the signal that firms receive about each customer, firms can classify customers into two segments: *perceived good-type* customers with a good signal, $s_i = g$, and *perceived bad-type* customers with a bad signal, $s_i = b$. This is the perceived market segmentation from the firms' perspectives. For example, if a person has purchased an energy-saving light bulb, the platform may perceive her as being interested in an environmentally

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[10]Consumers can sometimes identify the source of their dissatisfaction with a product. Our analysis can accommodate this situation as a limit case of our model, where one of the prior beliefs goes to 1 or 0. However, our focus is on other more general scenarios such as new product or infrequently purchased product categories where consumers have little experience.

[11]We consider typical advertising situations where advertisers use an accessible advertising network such as that of Google, Facebook or Amazon, which provide the same customer information to all advertisers. However, in some cases, it is possible that different firms may have access to different data using their own first-party data.

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sustainable product category in general (Summers et al., 2016). How informative the noisy signals are depends on the type and amount of customer data.[12]

We measure the informativeness of the signal by $\alpha > 0$, which allows a possibility of imperfect targeting, for example, due to insufficient customer information (e.g., the platform has no historic information for a new customer) or imperfect information processing technology (Chen et al., 2001). If the true category match-type of customer i is good ($m_i = 1$), firms receive a correct signal $s_i = g$ with probability α. Otherwise, with probability $1 - \alpha$, the platform provides a signal that is randomly drawn from the prior beliefs about customer types so that $s_i = g$ with probability μ_0 and $s_i = b$ with probability $1 - \mu_0$. Similarly, if the true match of consumer i is bad ($m_i = 0$), firms receive a correct signal $s_i = b$ with probability α and otherwise receive a random signal according to prior beliefs. Formally, the signal structure can be summarized as follows:

$$\Pr(s_i = g | m_i = 1) = \alpha + (1 - \alpha)\mu_0, \qquad \Pr(s_i = b | m_i = 1) = (1 - \alpha)(1 - \mu_0)$$
$$\Pr(s_i = b | m_i = 0) = \alpha + (1 - \alpha)(1 - \mu_0), \qquad \Pr(s_i = g | m_i = 0) = (1 - \alpha)\mu_0$$

$$(2)$$

A lower α implies noisier customer information, whereas a higher α means firms can almost perfectly identify each customer's type.[13] Therefore, parameter α captures the informativeness of signals about consumers' types, and thus, we refer to α as *targeting accuracy*.

Targeted advertising

Given the customer data with targeting accuracy $\alpha > 0$, two competing firms choose the extent of their advertising. In particular, firm j of private quality type q_j chooses its *advertising intensity* for two segments of consumers: the perceived good-type ($s_i = g$), and the perceived bad-type ($s_i = b$). More formally, the firm's advertising strategy is defined as a mapping $\sigma_j(q) = (\sigma_j^g(q), \sigma_j^b(q))$, where $\sigma_j^s(q) \in [0, 1]$ denotes the fraction of consumers with signal $s \in \{g, b\}$ to be reached by the firm's advertising. For example, in an extreme case of $\sigma_j^g(q) = 1$ and $\sigma_j^b(q) = 0$, the firm sends advertising to all consumers perceived as good-type and to none of those perceived as bad-type.

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[12]It is reported that while the precision of data in most platforms can be anywhere between 10% and 20% (e.g., even gender is usually only 75% accurate), targeting accuracy in Facebook can be an order of magnitude better than anywhere else apart from a few exceptions such as Google Search (*Forbes*, "How Accurate is Marketing Data?" on July 5, 2017).

[13]It is also worth noting that this signal structure preserves the mean in the sense that the unconditional distribution of signals (s_i) has the same mean as consumers' true category match-type (m_i): $\Pr(s_i = g) = \mu_0 \cdot \Pr(s_i = g | m_i = 1) + (1 - \mu_0) \cdot \Pr(s_i = g | m_i = 0) = \mu_0$.

Each firm's actual advertising level is observed by neither the other firm nor the consumers. However, consumers have rational expectations about each firm's equilibrium advertising strategy, $\sigma_j^*(q) = (\sigma_j^{g*}(q), \sigma_j^{b*}(q))$. Hence, it is important to distinguish notation for firm j's chosen advertising level, $\widetilde{\sigma}_j = (\widetilde{\sigma}_j^g, \widetilde{\sigma}_j^b)$, and that for the firm's equilibrium advertising strategy, σ_j^*. Then, a fraction $\widetilde{\sigma}_j^g$ of perceived good-type consumers of mass $\Pr(s_i = g) = \mu_0$ and a fraction $\widetilde{\sigma}_j^b$ of perceived bad-type customers of mass $\Pr(s_i = b) = 1 - \mu_0$ would receive an ad from firm j. The total cost of advertising is increasing and convex in the total amount of advertising, $\mu_0 \cdot \widetilde{\sigma}_j^g + (1 - \mu_0) \cdot \widetilde{\sigma}_j^b$. In particular, $c(\widetilde{\sigma}) = k \cdot (\mu_0 \cdot \widetilde{\sigma}^g + (1 - \mu_0) \cdot \widetilde{\sigma}^b)^2$, where $k > 0$ is a constant that captures the unit cost of advertising.

Consumers may receive an advertisement from both firms, just one firm, or no firm. Thus, there are four distinct segments of consumers belonging to different advertising states. Consumer i's advertising state is defined by θ^{a_A, a_B}, where $a_j \in \{0, 1\}$ indicates whether the consumer received advertising from firm j (denoted by $a_j = 1$) or not (denoted by $a_j = 0$). For example, $\theta^{1,1}$ represents the state in which the consumer received both firms' advertising; in states $\theta^{1,0}$ and $\theta^{0,1}$ she received only advertising from firm A and B, respectively, and in state $\theta^{0,0}$, no advertising was received. Then, the realized distribution over the set of advertising states is

$$\Pr(\theta_i^{1,1}) = \mu \tilde{\sigma}_A^g \tilde{\sigma}_B^g + (1 - \mu) \tilde{\sigma}_A^b \tilde{\sigma}_B^b, \qquad \Pr(\theta_i^{0,1}) = \mu(1 - \tilde{\sigma}_A^g) \tilde{\sigma}_B^g + (1 - \mu)(1 - \tilde{\sigma}_A^b) \tilde{\sigma}_B^b$$

$$\Pr(\theta_i^{1,0}) = \mu \tilde{\sigma}_A^g (1 - \tilde{\sigma}_B^g) + (1 - \mu) \tilde{\sigma}_A^b (1 - \tilde{\sigma}_B^b), \qquad \Pr(\theta_i^{0,0}) = \mu(1 - \tilde{\sigma}_A^g)(1 - \tilde{\sigma}_B^g) + (1 - \mu)(1 - \tilde{\sigma}_A^b)(1 - \tilde{\sigma}_B^b).$$

$$(3)$$

Consumer decisions and Timeline

The game proceeds in three stages. Stage 1 is the *advertising stage*. Each firm is endowed with its quality type q_j, drawn independently from $U[0,1]$. Firms receive a signal about each customer's type $s_i \in \{g, b\}$. Given firms' quality q_j, firms choose their levels of advertising $\widetilde{\sigma}_j^s$ for the *perceived* s-type segment ($s \in \{g, b\}$) with information accuracy $\alpha \in (0, 1)$ based on customer data. Each consumer receives advertising from either, both or none of the firms. Firms also charge prices (p_j), which we first assume to be exogenously fixed, and all firms charge the same price, $p_A = p_B = p$, where $p \leq 1$ so that the trade always occurs when $u_{ij} = 1$. We consider a market where prices of most products are similar, e.g., that of mobile apps at \$0.99 in our running example. We abstract away pricing issues by this assumption, which allows the paper to focus on the consumer inference triggered by targeted advertising. However, we also acknowledge that prices are not fixed in many situations. Thus, after we have fully characterized the consumer inference process, we relax this assumption and endogenize each firm's optimal pricing decision in our extension. For now, we assume that prices are fixed in the product category level.

Stage 2 is the *inference stage*, in which each consumer makes inferences based on her advertising state θ_i and decides whether to visit a firm and which firm to visit first. A consumer who receives an advertisement from a firm can visit the firm's website at no cost by simply clicking on an interactive link or banner. If a consumer receives ads from both firms ($\theta^{1,1}$), she randomly chooses to visit one firm first. Finally, if a consumer receives no ads ($\theta^{0,0}$), she remains unaware of the new product category and thus does not participate in the market. Once consumer i visits firm j, she learns her utility $u_{ij} \in \{0, 1\}$. However, she cannot separately observe her exact match for the category $m_i \in \{0, 1\}$, and the product value $v_j \in \{0, 1\}$, which is a function of the firm's unobserved quality q_j. On the other hand, if the consumer does not enjoy the product ($u_{ij} = 0$), she makes inferences about two dimensions: her match with the category, m_i, and product value, v_j. Depending on these inferences, she decides whether to further search for the other firm. Consumers' two-dimensional belief updating is influenced by firms' advertising strategy, consumer's advertising state, and targeting accuracy.

Stage 3 is the *search stage*, in which consumers decide whether to continue to search for the other firm at search cost t_i. It is important to note that consumers are unaware of a particular product category until they receive an advertisement (such as one for a mobile scanning app in our opening example). Therefore, they are unable to engage in an independent search for firms in the category in the beginning. Consumers become aware of the existence of such a product category only after receiving an ad and, therefore, can engage in a costly search for another firm. For example, a Google search will yield the competitor's identity. Thus, if a consumer is unaware of the product category, she cannot engage in any product search. Still, consumers have a common understanding that the average firm quality in any given product category follows a prior distribution.

Consumers have heterogeneous search costs drawn independently from a uniform distribution with support $[T - \Delta, T + \Delta]$, where T captures the market-level average consumer search cost, and Δ represents the extent of heterogeneity in consumer search cost.[14] If the consumer decides to search for another firm, she incurs search costs irrespective of whether she has received an advertisement in Stage 1. Even if a consumer received ads from both firms, if she did not visit a firm at that time, she still has to incur extra time or effort to remember and find the prior advertisement that she had once overlooked.[15] Figure 1 depicts the entire sequence of the game.

We adopt the perfect Bayesian equilibrium as the solution concept, defined as follows: (1) each firm's advertising strategy $\sigma_j(q)$ maximizes its expected profit for a given quality $q_j \in [0, 1]$, given the other firm's advertising strategy and consumers' search and purchase decisions; (2) each consumer makes search and purchase

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[14]Throughout the paper, we assume that $T + \Delta > \frac{\mu_0}{3(3 - 2\mu_0)} - p$ to avoid an extreme case of everyone in the market engaging in the search for the second firm.

[15]The cost of searching beyond the prominent firm may differ depending on whether the consumer received an ad from it. The cost must be lower if she has received an advertisement in the past than in the alternate scenario. Nevertheless, we assume identical additional search costs in both cases for simplicity. We can relax this assumption, but the results are robust. What is crucial is the cost difference between the first and the subsequent visits. The subsequent visits are more costly than the rather effortless very first visit by clicking on a link. As long as there is a small additional cost associated with any subsequent visit (it can stem from time or effort involved in finding out the other ad or using a search engine such as Google), our results hold.
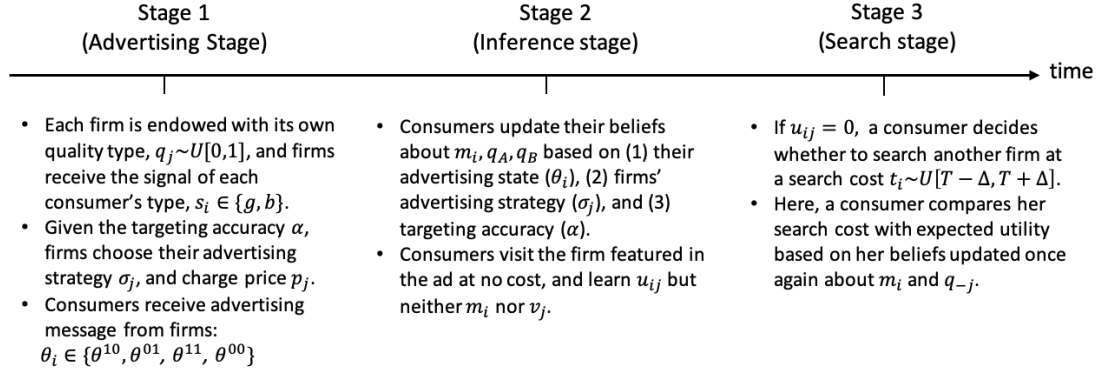
Figure 1: Timing of the Game

decisions optimally, given the firm's advertising strategy and the consumer's beliefs; and (3) the consumer's beliefs are updated according to Bayes' rule and are consistent with firms' advertising strategy. Here, we focus on the symmetric equilibrium in which both firms choose $\sigma^*(q) = \sigma_A^*(q) = \sigma_B^*(q)$ for any $q \in [0,1]$.

4   Analysis

We start by examining the consumers' rational inference process when they observe targeted advertising. With the understanding of this micro-process of consumers' inference, we analyze their search and purchase decisions. Next, we investigate the firms' advertising strategy, which, in turn, alters consumer inference. Finally, we derive the equilibrium outcomes, taking into account both consumers' inference and firms' optimal advertising strategy. Proofs of all results are presented in the Appendix.

4.1   Consumer Inference

Consumer i does not know her category match type (m_i) and each firm's quality type (q_j), and holds prior beliefs $\mu_0 = \Pr(m_i = 1)$ and $q_0 = \mathbb{E}[q]$. After she observes her advertising state $\theta_i^{a_A, a_B} \in \{\theta_i^{1,1}, \theta_i^{1,0}, \theta_i^{0,1}, \theta_i^{0,0}\}$, the consumer updates her beliefs about m_i and q_j based on each firm's advertising strategy σ_j, and targeting accuracy $\alpha > 0$.

Updating beliefs about own category-match type

Given firm j's advertising strategy $\sigma_j(q) = (\sigma_j^g(q), \sigma_j^b(q))$ for any given $q \in [0,1]$, the consumer's posterior belief about her type after realizing advertising state $\theta_i^{a_A, a_B}$ is as follows:

$$\Pr(m_i = 1|\theta_i^{a_A,a_B}) = \frac{\Pr(m_i = 1) \cdot \left[\sum\limits_{s \in \{g,b\}} \Pr(\theta_i^{a_A,a_B}|s) \cdot \Pr(s|m_i = 1) \right]}{\sum\limits_{m_i \in \{0,1\}} \Pr(m_i) \left[\sum\limits_{s \in \{g,b\}} \Pr(\theta_i^{a_A,a_B}|s) \cdot \Pr(s|m_i) \right]} \tag{4}$$

$\Pr(\theta_i^{a_A,a_B}|s)$ is the distribution of realized advertising state conditional on the noisy signal associated with the consumer ($s \in \{g,b\}$). It depends on each firm's advertising strategy, which is a function of its private quality type. However, consumers do not observe the firm's quality q_j and therefore account for the expected advertising strategy in updating beliefs. Let us define

$$\widehat{\sigma}_j^s := \mathbb{E}[\sigma_j^s(q)] = \int_0^1 \sigma_j^s(q)\, dq. \tag{5}$$

Then, the consumer's expected probability distribution over the advertising states, given signal s, is $\Pr(\theta_i^{1,1}|s) = \widehat{\sigma}_A^s \cdot \widehat{\sigma}_B^s$, $\Pr(\theta_i^{1,0}|s) = \widehat{\sigma}_A^s \cdot (1 - \widehat{\sigma}_B^s)$, $\Pr(\theta_i^{0,1}|s) = (1 - \widehat{\sigma}_A^s) \cdot \widehat{\sigma}_B^s$, and $\Pr(\theta_i^{0,0}|s) = (1 - \widehat{\sigma}_A^s)(1 - \widehat{\sigma}_B^s)$. Therefore, as we can see from (4), the consumer's posterior belief about the match type depends on the prior ($\mu_0 = \Pr(m_i = 1)$), firms' equilibrium advertising strategy through $\Pr(\theta_i^{a_A,a_B}|s)$, and targeting accuracy α.[16]

The next proposition characterizes the consumer's belief updating process about her match with the product category after receiving a targeted ad.

Proposition 1 (Posterior Beliefs about Consumer's Category Match-type) *Having been targeted, a consumer updates her beliefs about her category match-type more positively:* $\Pr(m_i = 1|a_j = 1) - \Pr(m_i = 1|a_j = 0) \geq 0$ *if and only if* $\widehat{\sigma}_j^g > \widehat{\sigma}_j^b$. *The marginal change in the posterior beliefs is increasing in targeting accuracy* α: $\frac{\partial\, [\Pr(m_i = 1|a_j = 1) - \Pr(m_i = 1|a_j = 0)]}{\partial\, \alpha} \geq 0$.

This proposition characterizes a necessary and sufficient condition under which being targeted enhances a consumer's beliefs about her category match-type m_i. As expected, this happens if the firm expends more significant efforts on the perceived good-type consumers than on the perceived bad-types. This marginal effect of being targeted on a consumer's beliefs is greater if targeting is more accurate.

Updating beliefs about a firm's quality

Whether a consumer receives firm j's targeted advertising also affects her beliefs about the firm's quality level and its product match. The posterior beliefs about firm j's unobserved quality depend on whether the consumer received an ad from the firm, $a_j \in \{0,1\}$. It is defined as follows:

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[16]The targeting accuracy influences belief updating in (4) through $\Pr(s|m_i)$, where $\Pr(s_i = g|m_i = 1) = \alpha + (1-\alpha) \cdot \mu_0$, and $\Pr(s_i = b|m_i = 0) = \alpha + (1-\alpha) \cdot (1 - \mu_0)$, as defined in (2).

$$h_j(q|a_j = 1) = \frac{\mu_0 \cdot \sigma_j^g(q) + (1 - \mu_0) \cdot \sigma_j^b(q)}{\int_0^1 \left(\mu_0 \cdot \sigma_j^g(y) + (1 - \mu_0) \cdot \sigma_j^b(y)\right) \, dy},$$

$$h_j(q|a_j = 0) = \frac{\mu_0 \cdot (1 - \sigma_j^g(q)) + (1 - \mu_0) \cdot (1 - \sigma_j^b(q))}{\int_0^1 \left(\mu_0 \cdot (1 - \sigma_j^g(y)) + (1 - \mu_0) \cdot (1 - \sigma_j^b(y))\right) \, dy}. \tag{6}$$

Note that consumers' inferences about firm $j's$ quality are not influenced by whether they received an advertisement from the other firm. The reason is that firms' quality types are independent, and therefore, consumers derive no additional information about firm j's type from the other firm's advertising strategy.

Proposition 2 (Posterior Beliefs about a Firm's Quality Type) *The posterior belief about a firm's quality $h_j(q|a_j)$ satisfies the monotone likelihood ratio property (MLRP): $\frac{h_j(q|a_j=1)}{h_j(q|a_j=0)}$ is increasing in q if and only if the total amount of advertising is increasing in q: $\frac{\partial}{\partial q}\left(\mu_0 \cdot \sigma_j^g(q) + (1 - \mu_0) \cdot \sigma_j^b(q)\right) > 0$.*

The *MLRP* implies that upon observing a targeted advertisement ($a_j = 1$), the consumer's posterior distribution over the firm's unobserved quality becomes more optimistic.[17] In other words, consumers will update their beliefs more positively if a firm advertises more when its quality is high.

Two-dimensional belief updating

Consumers react to a targeted advertisement based on firms' advertising strategy, prior beliefs, and targeting accuracy. After observing advertising, consumers update their beliefs about their match type with the product category and each firm's quality. Under quite general conditions, a greater targeting accuracy can lead to more optimistic posterior beliefs about both product category matching and firm's quality.

Figure 2 demonstrates two-dimensional belief updating.[18] The solid line represents the marginal effect of a targeted advertisement on a consumer's posterior beliefs about the category match, $\Pr(m_i|a_j = 1) - \Pr(m_i|a_j = 0)$. The dotted line represents the same for the firm's quality type, $\Pr(v_j = 1|a_j = 1) - \Pr(v_j = 1|a_j = 0)$. An important takeaway from the graph is that the relative effects of targeted advertising on these two different dimensions change depending on market conditions. First, Figure 2-(a) demonstrates that the prior belief about the consumer's category match μ_0 always has positive effects on the beliefs about the quality type, and its effect increases monotonically. However, the effect on the consumer's own category match is nonmonotonic because there is a ceiling and little room for its effect as μ_0 approaches one. Therefore, if μ_0 is small, targeted advertisements affect consumers' beliefs about their own category match more than those about the firm's quality type. However, this result can be reversed if μ_0 is large. It implies that for new or innovative product categories with a low μ_0, a targeted advertisement can stimulate consumers' interest mainly because it influences their

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[17]The *MLRP* implies that $h_j(q|a_j = 1)$ has first-order stochastic dominance over $h_j(q|a_j = 0)$.

[18]The figure shows posterior beliefs identified in equilibrium, characterized in Section 4.3.
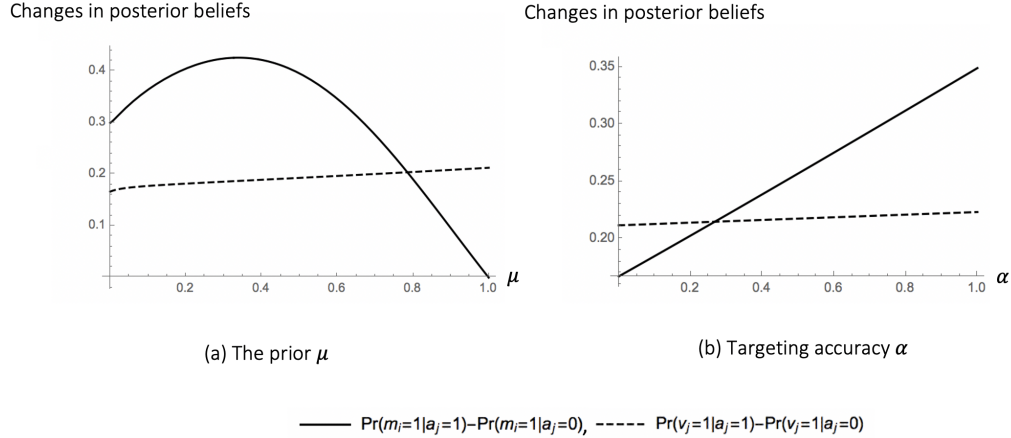
Changes in posterior beliefs   Changes in posterior beliefs

(a) The prior μ (b) Targeting accuracy α

—— $\Pr(m_i=1|a_j=1)-\Pr(m_i=1|a_j=0)$, - - - - $\Pr(v_j=1|a_j=1)-\Pr(v_j=1|a_j=0)$

Figure 2: Marginal effect of being targeting on the posterior beliefs as a function of (a) μ_0 (for $\alpha = 0.4$) and (b) α (for $\mu = 0.7$) (where $T = 0.6, \Delta = 0.2, p = 0.7, k = 0.6$)

beliefs that they may benefit from the product category in general. On the other hand, for a product category that appeals to a large segment of customers (i.e., μ_0 is already large), targeted advertisements can engage consumers in the product category by improving their inferences about product quality.

Second, Figure 2-(b) shows that targeting accuracy has positive effects on a consumer's beliefs about both the consumer's category type and the firm's quality. However, this positive effect is higher for the former. The targeting is based on the consumer's perceived category type. Therefore, targeting accuracy has direct effects on the beliefs about the category type, while its effect on the beliefs about the firm's quality is indirect and only through the firm's advertising strategy. That is, given a higher targeting accuracy, a firm of higher quality may invest more. Therefore, whether a consumer is targeted or not provides information about the firm's unobserved quality indirectly. This indirect effect is smaller than the direct effect on the beliefs about the category type.

4.2   Consumer Search and Demand

Each consumer, after observing her advertising state and making inferences in Stage 1, makes her first visit decision in Stage 2. If necessary, she makes her search decision beyond the first firm in Stage 3. Therefore, a firm's demand can come from two sources: *direct demand* (denoted by D_j^{Dir}) from those who first visit the firm and make purchases, and *indirect demand* (denoted by D_j^{Ind}) from those who visit the firm as the second firm.

To identify a symmetric equilibrium advertising strategy, without a loss of generality, we solve firm A's problem, given firm B's advertising strategy. We denote firm A's choice of advertising levels by $\widetilde{\sigma}_A = (\widetilde{\sigma}_A^g, \widetilde{\sigma}_A^b)$, given its private quality type. In equilibrium, both firms A and B choose their advertising levels based on their private quality types. However, from firm A's perspective, neither firm B's quality type nor its choice of advertising is observable. Instead, firm A forms an expectation over firm B's advertising level by averaging firm

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B's advertising strategy over the distribution of quality types, which we define as $\widehat{\sigma}_B := \mathbb{E}_q[\sigma_B(q)] = (\widehat{\sigma}_B^g, \widehat{\sigma}_B^b)$.

Direct Demand: A Costless Visit to the Prominent Firm

A consumer will visit firm A first if she receives that firm's advertisement: $\theta^{1,0}$ or $\theta^{1,1}$. If she receives ads from both firms $(\theta^{1,1})$, then she randomly chooses one firm to visit first because she is indifferent between the two options. Then, firm A's expected direct demand given firm B's expected advertising strategy is as follows:

$$
\begin{aligned}
D^{Dir}(\widetilde{\sigma}_A; \sigma_B) &= \Pr(m_i = 1)\Pr(v_A = 1) \sum_{s_i \in \{g,b\}} \Pr(s_i|m = 1)\big(\Pr(\theta^{1,0}|s_i) + \frac{1}{2}\Pr(\theta^{1,1}|s_i)\big) \\
&= \mu_0 \cdot q_A \bigg\{ \big(\alpha + (1-\alpha)\mu_0\big)\big(\sigma_A^g(1 - \widehat{\sigma}_B^g) + \frac{\widetilde{\sigma}_A^g \widehat{\sigma}_B^g}{2}\big) + (1-\alpha)(1-\mu_0)\big(\widetilde{\sigma}_A^b(1 - \widehat{\sigma}_B^b) + \frac{\widetilde{\sigma}_A^b \widehat{\sigma}_B^b}{2}\big) \bigg\}
\end{aligned}
\tag{7}
$$

Hence, firm A's expected direct demand increases in its advertising amount while decreasing in its competitor's expected advertising amount. This explains the competition between the firms for becoming the prominent firm to preempt demand before consumers search for the other firm (Armstrong et al., 2009). This prominence effect provides incentives for each firm to invest in costly advertising.

Lemma 1 (Incentives for Prominence) *Firm A's expected direct demand $D_A^{Dir}(\widetilde{\sigma}_A; \sigma_B)$ increases in its own advertising but decreases in the expected firm B's advertising:* $\frac{\partial D_A^{Dir}(\widetilde{\sigma}_A; \sigma_B)}{\partial \widetilde{\sigma}_A} > 0$, $\frac{\partial D_A^{Dir}(\widetilde{\sigma}_A; \sigma_B)}{\partial \widehat{\sigma}_B} < 0$.

Indirect Demand: A Costly Search Beyond the Prominent Firm

A consumer who visits firm B first may deem its product unsatisfactory, i.e., $u_{iB} = 0$. Then, the consumer updates her beliefs about m_i, and decides whether to search further by comparing her expected benefit from searching for additional firm A, $\mathbb{E}[u_{iA}|\theta^{0,1}, u_{iB} = 0] - p$, and her search cost t_i drawn from a uniform distribution on $[T - \Delta, T + \Delta]$.[19] This search decision depends on her updated beliefs about her match with the product category, defined in the term $\Pr(m_i = 1|\theta^{a_A,a_B}, u_{iB} = 0)$, and firm A's expected quality, which depends on whether she initially received an advertisement from firm A, defined as $\Pr(v_A = 1|\theta^{a_A,a_B}, u_{iB} = 0)$.[20] Therefore, the consumer's search decision boils down to the following rule:

$$
\mathbb{E}[\max\{0, u_{iA} - p\}|\theta^{0,1}, u_{iB} = 0] > t_i \Leftrightarrow \Pr(m_i = 1|\theta^{0,1}, u_{iB} = 0) \cdot \Pr(v_A = 1|\theta^{0,1}, u_{iB} = 0)(1 - p) > t_i \tag{8}
$$

More specifically, the consumer's belief updating for her own category match-type m_i follows Bayes' rule:

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[19]To ensure that some consumers engage in searching beyond the first firm, the price and the (lower bound of) consumer search cost must be sufficiently low, i.e., the condition $T - \Delta < \frac{1}{3} \cdot \frac{\alpha + (1-\alpha)\mu_0}{\alpha + (1-\alpha)\mu_0 + 3(1-\alpha)(1-\mu_0)}$ must hold.

[20]There can be two distinct initial advertising states that can lead to firm A's indirect demand: $\theta^{0,1}$ and $\theta^{1,1}$. Because the analyses of the two cases are similar, only the former case is presented in the main text.

$$\Pr(m_i = 1 | \theta^{0,1}, u_{iB} = 0) = \frac{\Pr(m_i = 1) \cdot \sum_{s_i \in \{g,b\}} \Pr(s_i | m_i = 1) \cdot \Pr(\theta^{0,1} | s_i) \cdot \Pr(u_{iB} = 0 | m_i = 1)}{\sum_{m_i \in \{0,1\}} \Pr(m_i) \sum_{s_i \in \{g,b\}} \Pr(s_i | m_i) \cdot \Pr(\theta^{0,1} | s_i) \cdot \Pr(u_{iB} = 0 | m_i)}. \qquad (9)$$

This is similar to equation (4) with one extra detail that $u_{iB} = 0$. The fact that the consumer is unsatisfied with firm B's product can lead to more pessimistic beliefs about the product category match. However, because she was targeted, her beliefs can still be sufficiently positive to engage in further searching for firm A if targeting is accurate.

The consumer's belief about firm A's private quality also affects her decision on whether to further search for the firm after being dissatisfied with firm B. Again, firm A's quality affects its own strategy but not firm B's strategy. Therefore, the posterior belief about firm A's unobserved quality is determined by whether a consumer received an advertisement from the firm, which is characterized in equation (6). In particular, for $\theta^{0,1}$ the posterior belief corresponds to $h_j(q|a_j = 0) = \frac{\mu_0 \cdot (1 - \sigma_j^g(q)) + (1 - \mu_0) \cdot (1 - \sigma_j^b(q))}{\int_0^1 \left(\mu_0 \cdot (1 - \sigma_j^g(y)) + (1 - \mu_0) \cdot (1 - \sigma_j^b(y)) \right) dy}$ from equation (6) because the consumer did not receive an advertisement from firm A.

Given these posterior beliefs, a consumer is more likely to search if her beliefs are more positive because, for instance, targeting is more accurate, or if the search cost t_i is smaller. As t_i is drawn from a uniform distribution on $[T - \Delta, T + \Delta]$, the consumer will search for firm A with probability

$$\Pr(\mathbb{E}[\max\{0, u_{iA} - p\} | \theta^{0,1}, u_{iB} = 0] > t_i) = \max\{0, \frac{\mathbb{E}[\max\{0, u_{iA} - p\} | \theta^{0,1}, u_{iB} = 0] - (T - \Delta)}{2\Delta}\}. \qquad (10)$$

This probability is decreasing in T. It implies that when the market-level average consumer search cost is higher, each consumer is less likely to search beyond the prominent firm.

Combining indirect demand generated through two advertising states $\theta^{0,1}$ and $\theta^{1,1}$, we obtain the total expected indirect demand

$$D_A^{Ind}(\widetilde{\sigma}_A; \sigma_B) = \mu_0 q_A \sum_{s_i \in \{g,b\}} \Pr(s_i | m_i = 1) \bigg\{ \Pr(\theta^{0,1} | s_i) \cdot \Pr(v_B = 0 | \theta^{0,1}) \cdot \Pr\left(\mathbb{E}[\max\{0, u_{iA} - p\} | \theta^{0,1}, u_{iB} = 0] > t_i \right)$$
$$+ \frac{\Pr(\theta^{1,1} | s_i)}{2} \cdot \Pr(v_B = 0 | \theta^{1,1}) \cdot \Pr\left(\mathbb{E}[\max\{0, u_{iA} - p\} | \theta^{1,1}, u_{iB} = 0] > t_i \right) \bigg\}.$$

Similarly, we can compute firm B's expected indirect demand from firm A's perspective, $D_B^{Ind}(\sigma_B; \widetilde{\sigma}_A)$. Then, we can show that the more firm A invests in advertising, the greater firm B's expected indirect demand. While firm A's advertisement may induce more consumers to visit the firm first, they may search for firm B if they are unsatisfied with firm A's product. Hence, firm A's advertising investment increases the number of consumers searching for firm B, which captures the extent of the free-riding effect from advertising spillover.

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Lemma 2 (Incentive to Free-ride) *In a symmetric equilibrium where $\sigma_j^* = \sigma^*$ for all $j \in \{A, B\}$, the indirect demand increases in the competitor's advertising level:* $\frac{D_B^{Ind}(\sigma_B; \widetilde{\sigma}_A)}{\widetilde{\sigma}_A} > 0$ *and* $\frac{D_A^{Ind}(\sigma_A; \widetilde{\sigma}_B)}{\widetilde{\sigma}_B} > 0.$

Given the exogenous price $p_A = p_B = p$, the total expected revenue is a function of the total expected demand. It is the sum of direct and indirect demand, characterized in (7) and (11), respectively. Therefore, firm A's expected profit, given the firm's choice of advertising level $\widetilde{\sigma}_A = (\widetilde{\sigma}_A^g, \widetilde{\sigma}_A^b)$ and firm B's advertising strategy $\sigma_B = (\sigma_B^g, \sigma_B^b)$ is the total expected revenue less the total advertising cost:

$$\mathbb{E}\Pi_A(\widetilde{\sigma}_A; \sigma_B) = p \cdot \left(D_A^{Dir}(\widetilde{\sigma}_A; \sigma_B) + D_A^{Ind}(\widetilde{\sigma}_A; \sigma_B) \right) - c(\widetilde{\sigma}_A) \tag{11}$$

4.3 Optimal advertising strategies

Benchmark: Non-targeted advertising without using customer information

We start our analysis with a benchmark case of non-targeted advertising. In contrast to the case of targeted advertising, consumers do not update beliefs about their own category match-type. Therefore, this benchmark helps us isolate the role of consumer inferences based on the mere fact that consumers were targeted beyond the simple awareness effect of advertising.

We define non-targeted advertising as the case of firms committing to not condition their advertising strategy based on customer information. Therefore, in our setting, non-targeted advertising is possible even if $\alpha > 0$ as long as the firm can commit to ignoring customer information. Non-targeted advertising does not coincide with the case of $\alpha = 0$.[21] Under non-targeted advertising, each firm j's advertising strategy $\sigma_j^{non}(q)$ is a simple mapping from its quality to the fraction of all consumers who will receive advertising. Therefore, firms send advertisements to the same fraction of consumers in both types, $\sigma^g = \sigma^b$. As ads are not targeted, consumers do not update their beliefs about their match with the product category. Instead, they only update their beliefs about the firm's quality. We now characterize the equilibrium advertising strategy under non-targeted advertising.

Proposition 3 (Benchmark: Non-targeted Advertising) *Under non-targeted advertising, if the unit cost of advertising is sufficiently high $(k > \overline{k}^{non})$, the unique symmetric equilibrium is characterized by $\sigma^{non*}(q) = \lambda^{non} \cdot q$, where $\lambda^{non} \in (0, 1)$. Therefore, in equilibrium, upon receiving any advertisement, a consumer makes a more positive inference about the advertiser's unobserved quality.[22]*

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[21]We will show that under the targeted advertising equilibrium, firms advertise only to perceived good-type customers (Proposition 4). Therefore, even if $\alpha = 0$ (where customer information is completely uninformative), firms can still choose to send advertising to a fraction μ_0 of consumers perceived to be good-type in the case of targeted advertising. On the other hand, in non-targeted advertising, firms will choose the advertising intensity for the entire market without restricting their targeting to the perceived good-type consumers. Therefore, the key to the distinction between the targeted and non-targeted advertising is whether or not the firms commit to conditioning their advertising strategy on their signals of consumer information.

[22]More precisely, the exact value of λ^{non} depends on the model primitives of μ_0, T, k, as well as p and Δ. Comparative

This proposition states an important point that in equilibrium, advertising intensity (i.e., the fraction of consumers reached by advertising) is linearly increasing in firm's quality. This implies that a higher-quality firm advertises more aggressively than a lower-quality firm. Therefore, upon receiving non-targeted advertising, a consumer rationally infers that the advertising firm is more likely of higher quality. On the other hand, because the advertising is not based on customer information, a consumer does not make inference about her category match-type.

Targeted advertising using customer information

In the case of targeted advertising, each firm sends targeted advertising based on customer information. Firm j chooses its advertising intensity based on its quality q_j: $\sigma^g(q_j)$ and $\sigma^b(q_j)$. We consider firm A's decision in equilibrium analysis without a loss of generality. To identify the conditions of a symmetric equilibrium, we differentiate the profit function in equation (11) with respect to $\widetilde{\sigma}_A^g$ and $\widetilde{\sigma}_A^b$:

$$\frac{\partial \mathbb{E}\Pi_A(\widetilde{\sigma}_A; \sigma^*)}{\partial \widetilde{\sigma}_A^g} = \underbrace{\frac{\partial D_A^{Dir}}{\partial \widetilde{\sigma}_A^g}}_{\text{prominence}} + \underbrace{\frac{\partial D_A^{Ind}}{\partial \widetilde{\sigma}_A^g}}_{\text{free-riding}} - \frac{\partial c(\widetilde{\sigma})}{\partial \widetilde{\sigma}_A^g}, \qquad \frac{\partial \mathbb{E}\Pi_A(\widetilde{\sigma}_A; \sigma^*)}{\partial \widetilde{\sigma}_A^b} = \underbrace{\frac{\partial D_A^{Dir}}{\partial \widetilde{\sigma}_A^b}}_{\text{prominence}} + \underbrace{\frac{\partial D_A^{Ind}}{\partial \widetilde{\sigma}_A^b}}_{\text{free-riding}} - \frac{\partial c(\widetilde{\sigma})}{\partial \widetilde{\sigma}_A^b}, \tag{12}$$

where $\frac{\partial c(\widetilde{\sigma})}{\partial \widetilde{\sigma}_A^g} = -2k \cdot \mu_0 \cdot (\mu_0 \widetilde{\sigma}_A^g + (1-\mu_0)\widetilde{\sigma}_A^b)$, $\frac{\partial D_A^{Dir}}{\partial \widetilde{\sigma}_A^g} = p \cdot \mu_0 (\alpha + (1-\alpha)\mu_0) \cdot q_A (1 - \frac{\widehat{\sigma}^g}{2})$, $\frac{\partial D_A^{Ind}}{\partial \widetilde{\sigma}_A^g} = -p \cdot \mu_0 (\alpha + (1-\alpha)\mu_0) \cdot$ $q_A ((1 - \mathbb{E}[q_B|\theta^{a_A}, 1]) \cdot (\widehat{\sigma}^g \cdot \max\{0, \frac{\mathbb{E}[\max\{0, u_{iA}-p\}|\theta^{0,1}, u_{iB}=0] - (T-\Delta)}{2\Delta}\} - \frac{\widehat{\sigma}^g}{2} \cdot \max\{0, \frac{\mathbb{E}[\max\{0, u_{iA}-p\}|\theta^{1,1}, u_{iB}=0] - (T-\Delta)}{2\Delta}\})$.

All the expressions in $\frac{\partial \mathbb{E}\Pi_A(\widetilde{\sigma}_A; \sigma^*)}{\partial \widetilde{\sigma}_A^b}$ are similarly defined in (18) in the Appendix.

The firm balances the benefit of advertising, considering the trade-off between the prominence effect $(\frac{\partial D_A^{Dir}}{\partial \widetilde{\sigma}_A^g})$ and the free-riding effect $(\frac{\partial D_A^{Ind}}{\partial \widetilde{\sigma}_A^g})$ against the cost of advertising $(\frac{\partial c(\widetilde{\sigma})}{\partial \widetilde{\sigma}_A^g})$. As (12) shows, the firm has an incentive to invest more in targeted advertising because it will induce more consumers to consider the firm first and capture a greater direct demand (the positive prominence effect in Lemma 1). On the other hand, the firm has less incentive to invest because some of consumers who first consider its competitor will eventually search for the firm (free-riding). In other words, indirect demand is decreasing in the firm's advertising (Lemma 2).

The prominence and free-riding effects in targeted advertising are notably different from those in non-targeted advertising because of consumer inferences from targeting, which, in turn, influence a consumer's search decisions. Before analyzing this distinction between targeted and non-targeted advertising, we characterize an equilibrium under targeted advertising.

Proposition 4 (Equilibrium Targeting Strategy) *Under targeted advertising, if the unit cost of advertising k is sufficiently large, a symmetric equilibrium characterized by $\sigma^*(q) = (\sigma^{g*}(q), \sigma^{b*}(q)) = (\lambda^{tar} \cdot q, 0)$ exists for some constant $\lambda^{tar} \in (0, 1)$. Upon receiving an ad, a consumer makes a more positive inference both about*

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results for λ^{non} are presented in the Online Technical Appendix, omitted here for brevity. Moreover, the exact expression of $\overline{k}^{non}$ is provided in the proof of Proposition 3 in the Appendix.

her unknown category match and the advertiser's unobserved quality.

If the unit cost k of advertising is sufficiently high, then in equilibrium, firms send their ads only to the perceived good-type consumers and none to the perceived bad-types. This is intuitive, given that, holding all else constant, a perceived good-type consumer is more valuable than a perceived bad-type consumer. Therefore, if k is large, firms in equilibrium will first cover the segment of perceived good-type consumers before the perceived bad-types. Otherwise, there is little reason for firms to restrict their advertising efforts only to a subset of all customers (i.e., the perceived good-types).

Similar to the case of non-targeted advertising, the equilibrium targeting strategy is characterized by an increasing linear function of each firm j's private quality type q_j. A higher-quality firm invests more aggressively in targeted advertising, thus satisfying the MLRP condition in Proposition 2. Consequently, consumers make more positive inferences about the firm's quality upon receiving the firm's advertisement.

However, in contrast to non-targeted advertising, firms concentrate their advertising efforts on the perceived good-types. Therefore, upon being targeted, a consumer makes more optimistic inferences about her match with the product category. As targeting accuracy α increases, consumers will hold more optimistic beliefs. With the more optimistic updated beliefs, consumers may engage in costly search beyond their prominent firm if they are dissatisfied with it.

The next proposition compares the indirect demand, which captures the extent of consumer search behavior, in non-targeted and targeted advertising cases. This approach allows us to examine the effect of consumer inferences on consumer search, which is triggered by the mere fact that a consumer is targeted.

Proposition 5 (Effect of Targeted Advertising beyond Awareness) *In equilibrium, there exists a threshold $\overline{\alpha} \in (0,1)$ such that for all $\alpha \geq \overline{\alpha}$, the indirect demand is greater in the targeted advertising case than in the non-targeted advertising case: $D^{Ind} - D_o^{Ind} > 0$.*

While targeted consumers in the case of targeted advertising hold optimistic beliefs about product category match, there is no consumer belief updating about the product category match resulting from receiving an advertisement in the case of non-targeted advertising. Thus, a comparison of indirect demand between the cases of targeted and non-targeted advertising demonstrates the incremental effect of consumer inference from being targeted on consumer search beyond the level of the mere awareness effects that advertising can cause. As the proposition demonstrates, greater targeting accuracy increases the effect of inference. Consumers will have more optimistic beliefs about product category match, which will increase the advertising spillover effect where consumers search beyond the first firm in the category.

The next proposition describes how the equilibrium level λ^{tar} of advertising depends on the key model parameters such as advertising cost (k), average search cost (T), and product price (p).

Proposition 6 (Comparative Statics) *The equilibrium intensity of targeted advertising decreases in the unit cost of advertising, but increases in the average search cost:* $\frac{\partial \lambda^{tar}}{\partial k} \leq 0$, $\frac{\partial \lambda^{tar}}{\partial T} \geq 0$. *Moreover, if k is sufficiently large, the intensity increases in product price:* $\frac{\partial \lambda^{tar}}{\partial p} \geq 0$.

As expected, firms reduce the amount of advertising in equilibrium if the unit cost k of advertising increases. Additionally, if T is high, consumers are less likely to search beyond the prominent firm. The reduced amount of consumer search reduces firms' incentives to free-ride on the competitor's advertising efforts. Therefore, both firms respond by investing more aggressively in targeted advertising, and hence, $\frac{\partial \lambda^{tar}}{\partial T} \geq 0$. Moreover, the exogenous price has an effect on λ^{tar} similar to that of the search cost in the sense that if p increases, a consumer will expect a lower benefit from additional search, which will reduce free-riding incentives in advertising. Furthermore, each unit of sales leads to greater revenue. Because of these two effects, firms opt for more advertising at higher prices, i.e., $\frac{\partial \lambda^{tar}}{\partial p} \geq 0$. [23] We are now ready to present our main result of this paper about how targeting accuracy affects the equilibrium level of advertising through consumer inference and search behaviors.

Proposition 7 (Effect of Targeting Accuracy on Advertising Amount) *If T is sufficiently large, firms' equilibrium advertising amount under competition increases in targeting accuracy* α *(*$\frac{\partial \lambda^{tar}}{\partial \alpha} > 0$*). Otherwise,* $\frac{\partial \lambda^{tar}}{\partial \alpha}$ *can be nonmonotonic. In particular,* λ^{tar} *first increases in* α *(*$\frac{\partial \lambda^{tar}}{\partial \alpha} > 0$*) and then decreases (*$\frac{\partial \lambda^{tar}}{\partial \alpha} \leq 0$*) when* α *becomes sufficiently high.*

The proposition provides insight into how targeting accuracy affects the firm's equilibrium advertising amount under competition. A higher targeting accuracy produces two opposing forces in terms of firms' *advertising incentives*. Firms can reach the right consumers for the product category with a higher probability, and therefore, each firms' advertising becomes more efficient. Hence, firms increase their investments in targeted advertising to become prominent. On the other hand, consumers who are dissatisfied with the prominent firm are more willing to search for the second firm because accurate targeting generates more positive inferences about their match type with the product category. Therefore, more precise targeting can induce more consumer search (Proposition 5), which in turn increases free-riding effects in advertising and thus reduces firms' incentives to advertise. An interplay between these *prominence* and *free-riding* effects can result in nonmonotonic effects of targeting accuracy on the equilibrium level of advertising. In particular, the free-riding effects become more pronounced for a large value of α because, if targeted, consumers make increasingly more positive inferences, which leads to more extensive consumer search behavior. Figure 3-(a) demonstrates this case where T is not too large so that there is a significant amount of consumer search beyond the prominent firm. In this case, the free-riding effects can be significant enough that if targeting accuracy is very high (α is close to 1), the firms' equilibrium level of advertising may decrease ($\frac{\partial \lambda^{tar}}{\partial \alpha} \leq 0$). That is, a higher targeting accuracy may result in a greater extent of free-riding in advertising, which curbs firms' equilibrium investment in advertising.

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[23]This is the effect of price under exogenous pricing. Section 5 analyzes an extension to firms' endogenous pricing.

<table>
</table>

(a) A Small T: $T = 0.1$ *(b)* A Large T: $T = 0.4$
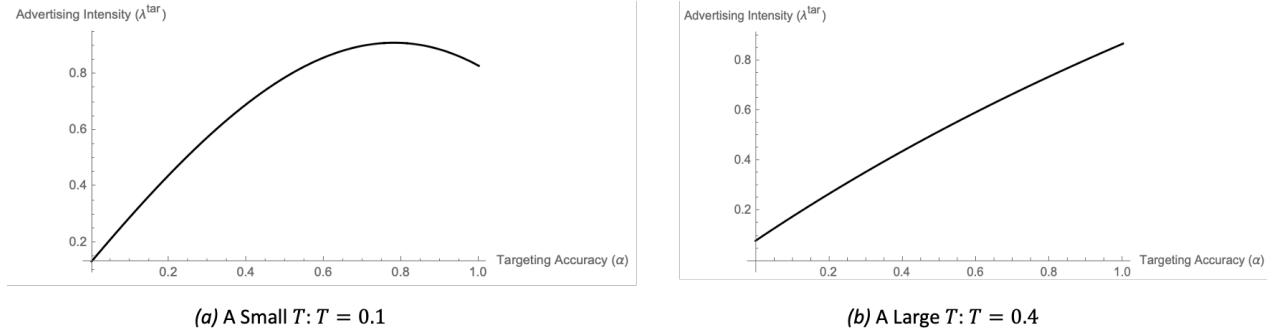
Figure 3: Advertising Reach under a Small vs. Large Search Cost

However, if T is sufficiently large, consumers do not search as much, which reduces free-riding effects. Therefore, the prominence effect dominates. Attracting consumers to visit the firm first and preempting more demand become more important. Thus, as targeting becomes more accurate, firms invest aggressively in targeted advertising, i.e., $\frac{\partial \lambda^{tar}}{\partial \alpha} \geq 0$. Figure 3-(b) demonstrates this case.[24]

4.4 Reach vs. Accuracy in Advertising: a Profit Analysis

Given the equilibrium advertising strategy under targeted advertising, $\sigma^*(q) = \left(\lambda^{tar}q, 0\right)$, firms send ads only to a fraction μ_0 of perceived good-type consumers in the market. Then, the total amount of advertising is $\left(\mu_0 \cdot \lambda^{tar} \cdot q\right)$. Therefore, $\mu_0 \cdot \lambda^{tar}$ can be a useful proxy for the total *advertising reach* under targeted advertising in equilibrium. Comparing this with the benchmark case $1 \cdot \lambda^{non}$ of non-targeted advertising, where advertising is sent to the entire market of size 1, gives the following result:

Proposition 8 (Comparison: Advertising Reach) *Suppose k is sufficiently large. When the targeting accuracy α is low, the advertising reach under non-targeted advertising is greater than targeted advertising $(\lambda^{non} > \mu_0 \cdot \lambda^{tar})$. However, as α becomes sufficiently high, there are two cases:*

1. *If T is sufficiently large, the advertising reach is greater under targeted advertising than non-targeted advertising $(\mu_0 \lambda^{tar} > \lambda^{non})$.*

2. *If T is not sufficiently large, the reach of targeted advertising can be smaller than the non-targeted advertising $(\mu_0 \lambda^{tar} < \lambda^{non})$.*

This proposition addresses the trade-offs between advertising reach and targeting accuracy, a topic of an ongoing debate in the industry, by comparing the advertising reach in cases of targeted and non-targeted advertising. Under targeted advertising, firms concentrate their advertising efforts on a subset of the entire market (i.e., that of perceived good type consumers of size μ_0) for $\alpha > 0$ as long as k is sufficiently large (Proposition

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[24]We use other parameter values of $\mu_0 = 0.075, p = 0.15, \Delta = 0.02$, and $k = 0.55$ and $k = 0.9$ for small and large T, respectively.

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
Electronic copy available at: https://ssrn.com/abstract=3688258

4). Firms only cover the perceived good-type consumers with intensity λ^{tar} even if α is low, resulting in an advertising reach of $\mu_0 \cdot \lambda^{tar}$. On the other hand, in the non-targeting case, firms commit to not conditioning their advertising strategy on customer information. Thus, firms engage in randomized ads by pre-committing to intensity λ^{non} of advertising for the entire market, irrespective of α, thus yielding advertising reach of $1 \cdot \lambda^{non}$. Therefore, holding the advertising intensity constant, the advertising reach should be greater under non-targeted advertising than under targeted advertising. This result turns out to be true if α is sufficiently small so that the benefit of targeting is minimal, leading to comparable intensity of λ^{non} and λ^{tar}. However, as targeting accuracy becomes higher (a higher α), firms invest more aggressively on targeted advertising, and λ^{tar} can far exceed λ^{non}. Consequently, the equilibrium advertising reach can be greater in targeted advertising than non-targeting case depending on the tradeoffs between prominence and free-riding effects. It has more nuanced effects on the equilibrium amount of targeted advertising (Proposition 7).

First, if T is large, then the extent of consumer search beyond the prominent firm is small, thus reducing free-riding incentives in advertising. So, firms invest more in advertising as targeting accuracy becomes higher. Then, although by construction firms focus on a subset of the entire customers, i.e., the perceived good-type consumers (μ_0 portion of entire population), surprisingly firms may invest more on targeted advertising than in the case of non-targeted advertising as targeting accuracy becomes sufficiently high. In this case, firms need not trade off between reach and targeting accuracy. A higher accuracy indeed leads to greater advertising reach.

On the other hand, if T is not large, a large extent of consumer search induces free-riding incentives in advertising, and the equilibrium level of targeted advertising λ^{tar} can be nonmonotonic in targeting accuracy (Proposition 7). In particular, the free-riding effects can be significant so that if targeting accuracy is very high (α close to 1), firms' equilibrium level of advertising in targeted advertising (and the equilibrium reach of advertising) may fall below the level in the non-targeting case. This result demonstrates a trade-off between targeting accuracy and advertising reach, which endogenously arises through competition in advertising, taking into account consumers' inferences from being targeted and their subsequent search decisions. Given the current proposition, we can now compare the firms' profits in targeted and non-targeted advertising scenarios.[25]

Corollary 1 *If T is sufficiently large, $\mathbb{E}\Pi^{tar*}(q) \geq \mathbb{E}\Pi^{non*}(q)$ if and only if $\alpha \geq \widehat{\alpha} = \frac{\mu_0 \cdot p}{8k - p(1-\mu_0)}$. The threshold level of $\widehat{\alpha}$ is increasing in μ_0 and p: $\frac{\partial \widehat{\alpha}}{\partial \mu_0} \geq 0$ and $\frac{\partial \widehat{\alpha}}{\partial p} \geq 0$.[26]*

If T is sufficiently large, the extent of consumer search beyond the prominent firm is small, thus reducing the

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[25]Note that we compare the equilibrium outcomes (advertising amount and profits) in two distinct scenarios: targeted and non-targeted advertising. One could consider a bigger game where firms first decide whether to adopt the targeting technology and then engage in competition through advertising. In our setting, endogenizing firms' technology adoption further complicates the analysis mainly because it creates another layer of consumer inference. Thus, we leave firms' optimal adoption of targeting technology for future research.

[26]This corollary does not consider all possible $T \geq \Delta$. For a more general consideration of all T, the analysis becomes algebraically complicated. Nevertheless, we identify a sufficient condition under which $\mathbb{E}\Pi^{tar*}(q) \geq \mathbb{E}\Pi^{non*}(q)$ holds. This sufficient condition (which involves other parameters such as k and μ_0) is presented in the Online Technical Appendix.

free-riding incentives in advertising. Hence, as targeting accuracy becomes high $(\alpha \geq \widehat{\alpha} = \frac{\mu_0 \cdot p}{8k - p(1-\mu_0)})$, firms compete aggressively in advertising, and the total reach of advertising increases (Proposition 8). However, if targeting accuracy is not high, firms cannot achieve enough advertising reach, so profits under targeted advertising fall below the profits under non-targeted advertising. Therefore, unless targeting accuracy is high enough, firms are better off forgoing customer information and instead engaging in non-targeted advertising.

We further characterize this threshold of α. First, as the prior μ_0 increases, the threshold level $\widehat{\alpha}$ rises, which implies that the interval of α that satisfied $\mathbb{E}\Pi^{tar*}(q) > \mathbb{E}\Pi^{non*}(q)$ shrinks. This result implies that firms in a mass-market product category with a broad appeal (represented by higher μ_0) are less likely to benefit from targeted advertising. In this case, firms want to achieve a greater advertising reach, making non-targeted advertising more appealing. On the other hand, for a product category that appeals to a narrow slice of the market (μ_0 is small), firms increasingly benefit more from targeted advertising. Additionally, as price increases, the region in which targeted advertising is more profitable shrinks. As discussed before, consumers facing a higher price expect less benefit from additional searching, which diminishes the distinction between targeted and non-targeted advertising. Therefore, targeted advertising becomes less appealing in comparison.

5 An Extension: Endogenous Pricing

So far, we have shown that consumers may make more positive inferences about their category match-types and advertising firm's quality type from a mere fact that they are targeted. In showing these results, we have assumed that prices are exogenously given. However, if such optimistic beliefs are held, firms might have incentives to adjust their prices to extract a surplus, which will affect consumers' search decisions. In this section, we study these nuanced issues that may arise from firms' strategic pricing by endogenizing pricing.

To make the pricing analysis more tractable, we modify the main model as follows. First, each firm's quality is assumed to be unknown to all players, even including the firm itself. This assumption implies that all players hold a common belief about each firm's quality throughout the game, which is $\mathbb{E}[q] = \frac{1}{2}$. This simplifies the analysis by eliminating consumers' inferences about each firm's quality while preserving the main forces induced by targeted advertising, i.e., the prominence effect and free-riding effects. Second, we modify the consumer's utility function by adding a different dimension of heterogeneity, which makes the pricing analysis more interesting and yields nontrivial equilibrium results. Recall that in the main model, the utility function in (1), $u_{ij} - p_j = \phi (m_i \cdot v_j) - p_j$, we normalize $\phi = 1$. Here, we modify the utility as follows:

$$u_{ij} - p_j = \phi_i (m_i \cdot v_j) - p_j \tag{13}$$

where ϕ_i, drawn from the standard uniform distribution $U[0,1]$, is the consumption utility of consumer i

conditional on having a good match with the product category and product j (i.e., $m_i \cdot v_j = 1$). We assume that initially ϕ_i is unknown to both consumer i and the firms. However, after a consumer visits a firm, she observes her ϕ_i and the price.[27]

We focus on a pricing strategy where each firm charges the same price in the entire market, both the perceived good-type and bad-type consumers. Hence, consumers do not make any inferences about their category match-type $m_i \in \{0, 1\}$ based on the observed prices. This allows us to focus on targeted advertising as the main channel of information transmission.[28] A consumer who visits firm B first realizes consumption utility $u_{iB} = \phi_i (m_i \cdot v_B)$ and price p_B. The consumer is unable to separately observe m_i and v_B. The consumer then forms inferences about m_i. Furthermore, in equilibrium the consumer has a correct expectation about firm A's price p_A^*. Based on these expectations, the consumer determines whether to search for firm A by paying the cost t_i. Since we have already captured the consumer heterogeneity by ϕ_i, we further simplify that $t_i = t$ for all consumers.[29] Similar to the consumer's search decision in the main model summarized in (8), a consumer who observes state θ and is dissatisfied with her first firm B ($u_{iB} = 0$) decides to search if and only if

$$\mathbb{E}[\max\{0, u_{iA} - p_A\}|\theta, u_{iB} = 0] > t \Leftrightarrow \Pr(m_i = 1|\theta, u_{iB} = 0) \cdot \frac{1}{2} \cdot \max\{0, \phi_i - p_A\} > t. \tag{14}$$

Considering consumers' belief updating and search decisions, which, in turn, depend on the firm's advertising and pricing strategies and its competitor's strategies, each firm makes its advertising and pricing decisions optimally. The following result characterizes a symmetric equilibrium with firms' endogenous pricing decisions.

Proposition 9 (Equilibrium with endogenous pricing) *If the unit cost k of advertising is sufficiently high and consumer search costs t are not too large, there exists a symmetric equilibrium with endogenous pricing. Both firms concentrate their advertising efforts on the perceived good types, i.e.,*

$$(\sigma^{g*}, \sigma^{b*}) = \left(\frac{(\alpha + (1-\alpha)\mu_0)p^*(1 - p^*)}{(\alpha + (1-\alpha)\mu_0)p^* \left(\frac{3(1-p^*)}{4} - \frac{t}{2\Pr(m_i=1|\theta, u_{iB}=0)} \right) + 2k\mu_0}, \ 0 \right)$$

and charge price $p^* = \left(3 - \frac{2t}{\Pr(m_i=1|\theta, u_{iB}=0)} \right)/5$.

The equilibrium advertising strategy is similar to that in the main model with exogenous pricing. Firms concentrate their advertising efforts on perceived good-type consumers. When the unit cost k of advertising is

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[27]Without this additional heterogeneity, a firm's pricing decision reverts to a trivial two-point case – either one or zero when consumers observe a firm's price after visiting the firm. We show this result in our Online Technical Appendix, where we also analyze the case of firms preannouncing prices that consumers can observe before their visit.

[28]If there were a pure-strategy equilibrium in which firms charged the perceived good-type and bad-type consumers different prices, the price a consumer observes would provide perfect information about her perceived match-type. Therefore, there would be no role of targeted advertising as an information transmission channel. Moreover, unlike with targeted advertising, there would be no direct cost of deviating from any supposed equilibrium. Thus, each firm would have an incentive to deviate by charging even the bad types the good-type price, which destroys this equilibrium.

[29]This simplification is only for analytical convenience and does not affect the main results.

sufficiently high, firms find it more profitable to focus on the consumers who are more likely to enjoy the product category. Therefore, consumers rationally draw more positive inferences about their category match-types from the mere fact that they are targeted, which is the same as in the exogenous pricing case.

We are primarily interested in the effect of targeting accuracy on firms' equilibrium amount of advertising and profits. Our analysis of endogenous pricing, unlike that of exogenous pricing, reveals interesting nuanced effects of pricing. The next lemma presents a useful underlying mechanism for establishing those results.

Lemma 3 *The equilibrium price p^* is increasing in targeting accuracy α, i.e., $\frac{\partial p^*}{\partial \alpha} > 0$. Moreover, both the direct demand and the indirect demand decrease in price: $\frac{\partial D^{Dir}}{\partial p}, \frac{\partial D^{Ind}}{\partial p} < 0$.*

This lemma establishes two interlinked relationships: (1) the relationship between targeting accuracy and price and (2) the relationship between the price and direct and indirect demand. First, the lemma suggests that the equilibrium price increases in targeting accuracy α. The reason is that as targeting accuracy increases, targeted consumers make more positive inferences about their match with the product category (as captured by the posterior probability $\Pr(m_i = 1|\theta, u_{i,B} = 0)$). Then, firms are now able to adjust prices upward to extract more surplus from such optimistic beliefs.

Additionally, we find that both direct and indirect demands decrease in price in equilibrium for quite distinct reasons. Direct demand decreases in price because of the firm's *own price effect*. Among the consumers who visit the firm first, more of them decide not to buy if the firm's price is higher. Indirect demand decreases in price because of the *cross-price effect*. A consumer deciding whether to search for the second firm is less likely to search if the second firm's price is higher. Based on this chain of relationships, we further analyze the effect of targeting accuracy on the equilibrium amount of advertising and profits.

Proposition 10 (Endogenous pricing: targeting accuracy, advertising and profits)

1. *If k is sufficiently large, the equilibrium amount of advertising can be nonmonotonic in targeting accuracy, $\frac{\partial \sigma^{g*}}{\partial \alpha} > 0$.*

2. *If t is sufficiently large, the equilibrium profit monotonically increases in targeting accuracy $\frac{\partial \pi(\sigma^{g*}, p^*)}{\partial \alpha} > 0$.*

Intuitively, the amount of consumer search, or indirect demand, may decrease in targeting accuracy because firms will charge a higher price in equilibrium. This implies that under endogenous pricing, even if targeting accuracy is high, firms' strategic decision to charge a higher price may reduce the amount of consumer search, thus mitigating firms' free-riding incentives in targeted advertising. Therefore, firms' investment in targeted advertising monotonically increases in targeting accuracy accordingly: $\frac{\partial \sigma^{g*}}{\partial \alpha} > 0$. This finding is in stark contrast to the model with exogenous pricing, where the equilibrium amount of advertising can be nonmonotonic in targeting accuracy.

Despite this interesting effect of endogenous pricing on the equilibrium amount of advertising, the proposition further shows that targeting accuracy turns out to have a positive effect on the firms' equilibrium profits, which is consistent with the result under exogenous pricing. The reason is that firms can identify and reach perceived good-type consumers with higher precision. This direct effect is strong enough to offset other nuanced indirect effects of targeting accuracy on advertising in equilibrium. In particular, if the consumer search cost t is sufficiently high such that fewer consumers search beyond the first firm, the free-riding incentives in advertising are mitigated, and therefore, firms' profits will increase as the targeting accuracy increases.

6   Conclusions

In this paper, we analyze a model of competitive targeted advertising with a consumer model that captures the micro-process of consumers' inferences and search behaviors. Firms have access to customer data, which allows them to imperfectly identify whether each consumer will benefit from a product category. Consumers, uncertain about their own benefit from the product category, as well as each firm's unobserved quality type, make inferences about both unobservables based on the mere fact that such consumers are targeted by advertising.

We first analyze the exogenous pricing case and fully characterize an equilibrium in which firms focus their advertising efforts only on consumers who are, according to customer data, likely to be good matches with the product category. We identify conditions under which the equilibrium level of advertising is increasing in the firm's quality type. Therefore, upon being targeted, consumers rationally make inferences that they are more likely to benefit from the product category, and that the firm is more likely to be of higher quality. We also demonstrate that the increase in indirect demand creates advertising spillover beyond the level of the simple awareness effects of advertising. The mere fact that an advertisement was targeted to consumers affects consumers' rational inference process and makes consumers hold optimistic beliefs about the product category. Moreover, this inference becomes even stronger as targeting accuracy improves. On the one hand, a greater targeting accuracy improves the match between consumers and the product category, which may increase the likelihood that consumers will be satisfied with the prominent firm they first visit. This finding implies that greater targeting accuracy may increase the incentives for firms to advertise more aggressively to be the first firm in the category to preempt the demand, which is the prominence effect. On the other hand, should targeted consumers be dissatisfied with the first firm, they could still hold optimistic beliefs about the product category in general and are therefore more likely to seek better alternatives. Hence, one firm's investment in targeted advertising can stimulate consumers' interest in the product category but eventually benefit its competitor. This creates a free-riding effect in advertising, which dissuades firms from making costly advertising investments. Based on these two opposing forces, we show that the equilibrium level of advertising increases in targeting accuracy if consumer search cost is sufficiently high so that the prominence effect dominates the free-riding

concern. However, if consumer search cost is not high, and more consumers are likely to engage in searching, the equilibrium level of advertising can be nonmonotonic in targeting accuracy. Moreover, we find that firms may spend more on targeted advertising than in the case of non-targeted advertising. This is surprising because, by construction, targeted advertising focuses on a subset of the entire market of consumers who are more likely to have a good match with the product category. Therefore, the potential reach of targeted advertising is smaller than that of non-targeted advertising. Nevertheless, we show that if targeting is highly accurate, firms invest in advertising more intensively, which can increase firms' profits. However, if the accuracy is not sufficiently high, firms can be better off relinquishing customer data and instead engaging in non-targeted advertising.

Finally, we incorporate the firms' strategic choice of prices. Firms have incentives to adjust their prices upward to extract a surplus from consumers when the latter hold more optimistic beliefs, making the endogenous pricing issues more subtle. In contrast to the exogenous pricing case, firms facing consumers with more optimistic beliefs charge a higher price, which reduces consumer surplus from additional searching. Therefore, optimistic beliefs resulting from being targeted do not lead to more consumer searches. Despite this interesting effect of endogenous pricing on the equilibrium amount of advertising, we show that higher targeting accuracy under endogenous pricing increases firms' equilibrium profits, which is consistent with the result for exogenous pricing.

For the endogenous pricing case, we consider a model in which consumers only see a firm's price after visiting the firm's website and realizing the consumption utility. Therefore, there is no room for the role of price as a signaling device.[30] However, there would be other settings in which the price could signal quality if, for instance, there were some costs incurred by a low-quality firm in mimicking a high-quality firm. Nevertheless, practically, in a new product category of which consumers are initially unaware, we believe that information about fit and quality is more commonly communicated through advertising than through pricing. We hope that future research can adequately extend the current model and examine this important price signaling issue in the framework of consumer inference from targeting.

Our work highlights the importance of the consumer inference process. In particular, we model targeting accuracy as one of the key primitives and study carefully how it affects consumers' rational inference process, which, in turn, influences firms' competitive advertising strategy in equilibrium. Our results can yield important insights into trade-offs between advertising reach and targeting accuracy, a topic of an ongoing debate in the industry. In our model, we treat targeting accuracy as given exogenously by the customer data and targeting technology. A future study can extend this limitation and fully endogenize the accuracy as the outcome of consumers' privacy concerns, firms' investments in targeting technology, or a platform's decision to share customer information.

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[30]In an alternative model where price information is pre-announced in an advertisement, there could be price signaling. However, in our setting, there is no mechanism to prevent a low-quality firm from mimicking a higher quality firm's price. Therefore, even if prices were observable to consumers before they visited stores, prices could not signal quality in equilibrium. We show this result in our Online Technical Appendix.

Appendix

Proof of Proposition 1

We only need to identify conditions of $\Pr(m_i = 1|\theta^{1,a_B}) - \Pr(m_i = 1|\theta^{0,a_B}) > 0$ for $a_B \in \{0,1\}$. First, for $a_B = 1$, $\Pr(m_i = 1|\theta^{1,1}) - \Pr(m_i = 1|\theta^{0,1})$ is $\frac{\alpha \cdot \mu_0 \cdot (1-\mu_0)(\widehat{\sigma}_A^g - \widehat{\sigma}_A^b) \cdot \widehat{\sigma}_B^g \cdot \widehat{\sigma}_B^b}{\left(\mu_0 \cdot \widehat{\sigma}_A^g \cdot \widehat{\sigma}_B^g + (1-\mu_0)\widehat{\sigma}_A^b \cdot \widehat{\sigma}_B^b\right) \cdot \left(\mu_0 \left(1-\widehat{\sigma}_A^g\right)\widehat{\sigma}_B^g + (1-\mu_0)(1-\widehat{\sigma}_A^b)\widehat{\sigma}_B^b\right)}$, which is greater than zero if and only if $\widehat{\sigma}_A^g - \widehat{\sigma}_A^b > 0$. Similarly, for $a_B = 0$, $\Pr(m_i = 1|\theta^{1,0}) - \Pr(m_i = 1|\theta^{0,0})$ is

$\frac{\alpha \cdot \mu_0 \cdot (1-\mu_0)(\widehat{\sigma}_A^g - \widehat{\sigma}_A^b)(1-\widehat{\sigma}_B^g)(1-\widehat{\sigma}_B^b)}{\left(\mu_0 \cdot (1-\widehat{\sigma}_A^g)(1-\widehat{\sigma}_B^g) + (1-\mu_0) \cdot (1-\widehat{\sigma}_A^b)(1-\widehat{\sigma}_B^b)\right) \cdot \left(\mu_0 \left(1-\widehat{\sigma}_B^g\right)\widehat{\sigma}_A^g + (1-\mu_0)(1-\widehat{\sigma}_B^b)\widehat{\sigma}_A^b\right)}$, which is also greater than zero if and only if $\widehat{\sigma}_A^g - \widehat{\sigma}_A^b > 0$. By symmetry, this proves the first part of the proposition. Additionally, from both equations above, it is clear that the marginal improvement in advertising is increasing in targeting accuracy, α. ∎

Proof of Proposition 2

It is sufficient to show that $\frac{h_A(q|a_j=1)}{h_A(q|a_j=0)}$ is increasing in q. Note that $\frac{h_A(q|a_j=1)}{h_A(q|a_j=0)}$ is $\frac{\mu_0 \cdot \sigma^g(q) + (1-\mu_0) \cdot \sigma^b(q)}{\mu_0 \cdot (1-\sigma^g(q)) + (1-\mu_0) \cdot (1-\sigma^b(q))} \times$ $\frac{\int_0^1 \mu_0 \cdot (1-\sigma^g(y)) + (1-\mu_0) \cdot (1-\sigma^b(y))\, dy}{\int_0^1 \mu_0 \cdot \sigma^g(y) + (1-\mu_0) \cdot \sigma^b(y)\, dy}$. Here, only the first fraction depends on q, and therefore, the ratio of two posterior beliefs is increasing in q if and only if $\frac{\mu_0 \cdot \sigma^g(q) + (1-\mu_0) \cdot \sigma^b(q)}{\mu_0 \cdot (1-\sigma^g(q)) + (1-\mu_0) \cdot (1-\sigma^b(q))}$ is increasing in q. Moreover, it is easy to verify that $\frac{\partial}{\partial q}\left(\frac{h_j(q|a_j=1)}{h_j(q|a_j=0)}\right) \geq 0 \Leftrightarrow \mu_0 \cdot \frac{\partial \sigma^g(q)}{\partial q} + (1-\mu_0)\frac{\partial \sigma^b(q)}{\partial q} \geq 0$. ∎

Proof of Lemma 1

The result directly follows from differentiating equation (7) with respect to $\widetilde{\sigma}_A^g$ and $\widetilde{\sigma}_A^b$: $\frac{\partial D^{Dir}(\widetilde{\sigma}_A)}{\partial \widetilde{\sigma}_A^g} = \mu_0 \cdot q_A \cdot \left(\alpha + (1-\alpha)\mu_0\right) \cdot \left(1 - \frac{\widehat{\sigma}_B^{g*}}{2}\right) > 0$, $\frac{\partial D^{Dir}(\widetilde{\sigma}_A)}{\partial \widetilde{\sigma}_A^b} = \mu_0 \cdot q_A(1-\alpha)(1-\mu_0) \cdot \left(1 - \frac{\widehat{\sigma}_B^{b*}}{2}\right) > 0$. ∎

Proof of Lemma 2

Firm B's expected indirect demand from firm A's perspective is $D_B^{Ind}(\sigma^*; \widetilde{\sigma}_A) = D_B^{Ind,g}(\sigma^*; \widetilde{\sigma}_A) + D_B^{Ind,b}(\sigma^*; \widetilde{\sigma}_A)$, where $D_B^{Ind,g}(\sigma^*; \widetilde{\sigma}_A) = \mu_0 \cdot (1-q_A) \cdot (\alpha + (1-\alpha)\mu_0) \cdot \left(\widetilde{\sigma}_A^g \cdot \mathbb{E}[(1-\sigma^{g*}(q_B)) \cdot q_B] \cdot \max\{0, \frac{\mathbb{E}[\max\{0, u_{iB}-p\}|\theta^{1,0}, u_{iA}=0] - (T-\Delta)}{2\Delta}\} + \frac{\widetilde{\sigma}_A^g \cdot \mathbb{E}[\sigma^{g*}(q_B) \cdot q_B]}{2} \cdot \max\{0, \frac{\mathbb{E}[\max\{0, u_{iB}-p\}|\theta^{1,1}, u_{iA}=0] - (T-\Delta)}{2\Delta}\}\right)$ and $D_B^{Ind,b}(\sigma^*; \widetilde{\sigma}_A) = \mu_0 \cdot (1-q_A)(1-\alpha)(1-\mu_0) \cdot \left(\widetilde{\sigma}_A^b \cdot \mathbb{E}[(1-\sigma^{b*}(q_B)) \cdot q_B] \cdot \max\{0, \frac{\mathbb{E}[\max\{0, u_{iB}-p\}|\theta^{1,0}, u_{iA}=0] - (T-\Delta)}{2\Delta}\} + \frac{\widetilde{\sigma}_A^b \cdot \mathbb{E}[\sigma^{b*}(q_B) \cdot q_B]}{2} \cdot \max\{0, \frac{\mathbb{E}[\max\{0, u_{iB}-p\}|\theta^{1,1}, u_{iA}=0] - (T-\Delta)}{2\Delta}\}\right)$. Therefore, it is clear that $\frac{\partial D_B^{Ind}(\sigma^*; \widetilde{\sigma}_A)}{\partial \widetilde{\sigma}_A^g} > 0$ and $\frac{\partial D_B^{Ind}(\sigma^*; \widetilde{\sigma}_A)}{\partial \widetilde{\sigma}_A^b} > 0$. ∎

Proof of Proposition 3

Firm A's direct demand is $D_{o,A}^{Dir}(\widetilde{\sigma}_A; \sigma^{non*}, q_A) = \mu_0 \cdot q_A \cdot \widetilde{\sigma}_A \cdot (1 - \frac{\widehat{\sigma}^{non*}}{2})$. Firm A's indirect demand is

$$
\begin{aligned}
D_{o,A}^{Ind}(\widetilde{\sigma}_A; \sigma^{non*}) &= \mu_0 q_A \cdot \mathbb{E}_{q_B}^{non}[\sigma^{non*}(q_B)(1-q_B)]\left\{\frac{\widetilde{\sigma}_A}{2}\frac{\max\{0, \mathbb{E}^{non}[\max\{0, u_A - p\}|\theta^{1,1}, u_B = 0] - (T - \Delta)\}}{2\Delta}\right. \\
&\quad + (1-\widetilde{\sigma}_A)\frac{\max\{0, \mathbb{E}^{non}[\max\{0, u_A - p\}|\theta^{0,1}, u_B = 0] - (T - \Delta)\}}{2\Delta}\Bigg\}.
\end{aligned}
$$

The first-order condition $\frac{\partial\,\Pi_A(\widetilde{\sigma}_A;\sigma^{non*})}{\partial\,\widetilde{\sigma}_A}|_{\widetilde{\sigma}_A=\sigma^{non*}}=0$ holds for all $q_A\in[0,1]$:

$$2k\cdot\sigma^{non*}(q_A)=p\cdot\mu_0\cdot q_A\Big\{1-\frac{\widehat{\sigma}^{non*}}{2}-\mathbb{E}^{non}_{q_B}[\sigma^{non*}(q_B)(1-q_B)]\times$$
$$\big(\max\{0,\frac{\mathbb{E}^{non}[\max\{0,u_A-p\}|\theta^{0,1},u_B=0]-(T-\Delta)}{2\Delta}\}-\frac{1}{2}\max\{0,\frac{\mathbb{E}^{non}[\max\{0,u_A-p\}|\theta^{1,1},u_B=0]-(T-\Delta)}{2\Delta}\})\Big\} \tag{15}$$

For any given strategy σ^{non*}, the right-hand side is equal to some constant multiplied by q_A. Therefore, $\sigma^{non*}(q_A)\equiv\lambda^{non}\cdot q_A$ for some constant λ^{non}. To pin down the constant λ^{non}, we substitute $\sigma^{non*}(q)=\lambda^{non}\cdot q$ into (15). As quality types are drawn from a standard uniform distribution, we can compute the expressions as follows: $\widehat{\sigma}^{non*}=\lambda^{non}\int q\,dq=\frac{\lambda^{non}}{2}$, $\mathbb{E}^{non}[q_A|a_A=1]=\int h^{non}_A(x|a_A=1)\,dx=\frac{\int x^2\,dx}{\int y\,dy}=\frac{2}{3}$, $\mathbb{E}^{non}[q_A|a_A=0]=\int xh^{non}_A(x|a_A=0)\,dx=\frac{\int x(1-\lambda^{non}x)\,dx}{\int(1-\lambda^{non}y)\,dy}=\frac{3-2\lambda^{non}}{3(2-\lambda^{non})}$, and $\Pr(m_i=1|\theta^{1,1},u_B=0)=\Pr(m_i=1|\theta^{0,1},u_B=0)=\frac{\mu_0(1-\mathbb{E}^{non}[q|a_B=1])}{\mu_0(1-\mathbb{E}^{non}[q|a_B=1])+1-\mu_0}=\frac{\mu_0}{3-2\mu_0}$. The result of plugging these properties into the first-order condition can be expressed as $\Gamma^{non}(\lambda^{non})=0$, where $\Gamma^{non}(\cdot)$ is defined as follows:

$$\Gamma^{non}(\lambda)=p\mu_0\Big[1-\frac{\lambda}{4}-\frac{\lambda}{6}\Big(\max\{0,\frac{1}{2\Delta}(\frac{\mu_0}{3-2\mu_0}\frac{3-2\lambda^{non}}{3(2-\lambda^{non})}(1-p)-(T-\Delta))\}-\frac{1}{2}\cdot\max\{0,\frac{1}{2\Delta}(\frac{\mu_0}{3-2\mu_0}\frac{2}{3}(1-p)-(T-\Delta))\}\Big)\Big]-2k\lambda^{non}, \tag{16}$$

where $\lambda=\lambda^{non}$ solves $\Gamma^{non}(\lambda)=0$.

We still need to show the existence of $\lambda^{non}\in[0,1]$; to this end, we invoke the intermediate value theorem. At $\lambda=0$, most terms vanish, and $\Gamma^{non}(0)=p\cdot\mu_0>0$. For $\lambda\neq0$, the exact expression of $\Gamma^{non}(\lambda)$ depends on the value of the two maximum operators. Note that for $\lambda\in[0,1]$, we have $\frac{1}{3}\leq\frac{3-2\lambda}{3(2-\lambda)}\leq\frac{1}{2}$. Accordingly, we consider the following two possibilities:

- Case I ("no consumers search" case): if $T>\frac{2\mu_0(1-p)}{3(3-2\mu_0)}+\Delta$, then the second operator vanishes, and therefore, so does the first max operator, i.e., no consumers search. Then, at $\lambda=1$, $\Gamma^{non}(1)=p\cdot\mu_0\cdot\frac{3}{4}-2k=\frac{3p\cdot\mu_0}{4}-2k<0$ if and only if $k>\frac{p\cdot\mu_0}{2}\cdot\frac{3}{4}$. Then, by the intermediate value theorem, $\lambda^{non}\in(0,1)$ that solves $\Gamma^{non}(\lambda)=0$ exists. Moreover, it is clear that $\frac{\partial\,\Gamma^{non}(\lambda)}{\partial\,\lambda}<0$, and hence λ^{non} is unique.

- Case II ("some consumers search" case): if $\frac{2\mu_0(1-p)}{3(3-2\mu_0)}-\Delta<T<\frac{\mu_0(1-p)}{3(3-2\mu_0)}+\Delta$, the values of both max operators are nonzero (between 0 and 1), i.e., some consumers search. At $\lambda=1$, $\Gamma^{non}(1)=p\cdot\mu_0\big(\frac{3}{4}+\frac{T-\Delta}{24\Delta}\big)-2k<0$ if and only if $k>\frac{p\cdot\mu_0}{2}\cdot\big(\frac{3}{4}+\frac{T-\Delta}{24\Delta}\big)$. By the intermediate value theorem, the solution $\lambda^{non}\in(0,1)$ for $\Gamma^{non}(\lambda)=0$ exists. Moreover, $\frac{\partial^2\,\Gamma^{non}(\lambda)}{\partial\,\lambda^2}=\frac{p\cdot\mu_0^2}{9\Delta(2-\lambda)^3(3-2\mu_0)}>0$, so λ^{non} is unique.

Therefore, in both cases, a unique $\lambda^{non}\in(0,1)$ exists if k is sufficiently large. ∎

Proof of Proposition 4

First, we show the existence of an equilibrium where firms focus their advertising efforts exclusively on the perceived good-types, i.e., $\sigma^{b*}(q)=0$ and $\sigma^{g*}(q)=\lambda^{tar*}\cdot q$. It is necessary to show that $\frac{\partial\,\mathbb{E}\Pi_A(\widetilde{\sigma}_A;\sigma^*)}{\partial\,\widetilde{\sigma}^g_A}=0$ and $\frac{\partial\,\mathbb{E}\Pi_A(\widetilde{\sigma}_A;\sigma^*)}{\partial\,\widetilde{\sigma}^b_A}\leq0$. Considering equation (12), we divide $\frac{\partial\,\mathbb{E}\Pi_A(\widetilde{\sigma}_A;\sigma^*)}{\partial\,\widetilde{\sigma}^g_A}$ by μ_0:

$$\frac{\partial\mathbb{E}\Pi_A(\widetilde{\sigma}_A;\sigma^*)}{\partial\widetilde{\sigma}^g_A}=0\Leftrightarrow2k\big(\mu_0\widetilde{\sigma}^g_A+(1-\mu_0)\widetilde{\sigma}^b_A\big)=q_Ap\big(\alpha+(1-\alpha)\mu_0\big)\Big[1-\frac{\widehat{\sigma}^{g*}}{2}-\mathbb{E}_{q_B}[\sigma^{g*}(q_B)(1-q_B)]\tau(\sigma^*)\Big] \tag{17}$$

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where $\tau(\sigma^*) = \max\{0, \frac{\mathbb{E}[\max\{0,u_A-p\}|\theta^{0,1},u_B=0]-(T-\Delta)}{2\Delta}\} - \frac{1}{2} \cdot \max\{0, \frac{\mathbb{E}[\max\{0,u_A-p\}|\theta^{1,1},u_B=0]-(T-\Delta)}{2\Delta}\}$. In equilibrium, this first-order condition of equation (17) must hold for all values of q_A. Note that for any strategy $\sigma^{g*}(q)$, the following terms are all constants: $\widehat{\sigma}^{g*}$, $\mathbb{E}[\max\{0,u_A-p\}|\theta^{1,1},u_B=0]$ and $\mathbb{E}[\max\{0,u_A-p\}|\theta^{0,1},u_B=0]$. Therefore, the right-hand side in equation (17) is equal to some constant multiplied by q_A, which implies that the left-hand side must also be of the same form: $\sigma^{g*}(q) + \sigma^{b*}(q) \equiv \lambda \cdot q$ for some constant $\lambda \in [0,1]$.

Similarly, considering equation (12), we divide $\frac{\partial \mathbb{E}\Pi_A(\widetilde{\sigma}_A;\sigma^*)}{\partial \widetilde{\sigma}_A^b}$ by $1 - \mu_0$:

$$\frac{\partial \mathbb{E}\Pi_A(\widetilde{\sigma}_A;\sigma^*)}{\partial \widetilde{\sigma}_A^b} = -2k\big(\mu_0\widetilde{\sigma}_A^{g*} + (1-\mu_0)\widetilde{\sigma}_A^b\big) + q_A p \mu_0 (1-\alpha)\left[1 - \frac{\widehat{\sigma}^{b*}}{2} - \mathbb{E}_{q_B}[\sigma^{b*}(q_B)(1-q_B)]\tau(\sigma^*)\right]. \qquad (18)$$

For any $\widetilde{\sigma}_A^b \geq 0$, $\frac{\partial \Pi_A(\widetilde{\sigma}_A;\sigma^*,q_A)}{\partial \widetilde{\sigma}_A^b} \leq 0 \Leftrightarrow -2k\sigma^{g*}(q_A) + q_A p \cdot (1-\alpha) \leq 0$. Using $\sigma^{g*}(q_A) = \lambda^{tar} \cdot q_A$, we have $k \geq \frac{(1-\alpha)p}{2\lambda^{tar}}$. Thus, we have proven that $\sigma^{b*}(q) = 0$ and $\sigma^{g*}(q) = \lambda^{tar*} \cdot q$ if k is sufficiently high ($k \geq \frac{(1-\alpha)p}{2\lambda^{tar}}$).

Next, we still need to show the existence of such a constant $\lambda^{tar} \in [0,1]$. Using the equilibrium advertising strategy provides $\widehat{\sigma}^{g*} = \int_0^1 \lambda^{tar} q\, dq = \frac{\lambda^{tar}}{2}$, $\mathbb{E}[\sigma^{g*}(q)(1-q)] = \frac{\lambda^{tar}}{6}$, $\mathbb{E}[q_j|a_j=1] = \frac{2}{3}$, $\mathbb{E}[q_j|a_j=0] = \frac{3-2\mu_0\lambda^{tar}}{3(2-\mu_0\lambda^{tar})}$, and $\Pr(m_i=1|\theta^{0,1},u_B=0) = \Pr(m_i=1|\theta^{1,1},u_B=0) = \frac{\alpha+\mu_0(1-\alpha)}{\alpha+\mu_0(1-\alpha)+3(1-\mu_0)(1-\alpha)} = (\zeta)$. Substituting these properties into equation (17) makes the first-order condition become

$$
\begin{aligned}
0 = \Gamma(\lambda^{tar}) &= -2k\mu_0^2\lambda^{tar} + p\mu_0(\alpha+(1-\alpha)\mu_0)\Big[1 - \frac{\lambda^{tar}}{4} \\
&\quad - \frac{\lambda^{tar}}{6}\Big(\max\big\{0, \frac{1}{2\Delta}\big(\frac{3-2\mu_0\lambda^{tar}}{3(2-\mu_0\lambda^{tar})}\zeta(1-p) - (T-\Delta)\big)\big\} - \frac{1}{2}\max\big\{0, \frac{1}{2\Delta}\big(\frac{2\zeta(1-p)}{3} - (T-\Delta)\big)\big\}\Big)\Big].
\end{aligned}
$$

We can easily see that if $\lambda^{tar} = 0$, $\Gamma(0) = p \cdot \mu_0 \cdot (\alpha+(1-\alpha)\mu_0) > 0$ for all q. We invoke the intermediate value theorem again by computing $\Gamma(\lambda^{tar} = 1)$. Note that the exact expression of $\Gamma(\lambda^{tar})$ depends on the value of the two maximum operators, where for $\lambda^{tar} \in [0,1]$, we have $\frac{1}{3} \leq \frac{3-2\lambda^{tar}}{3(2-\lambda^{tar})} \leq \frac{1}{2}$. Therefore, the value of the second max operator is greater than or equal to that of the first max operator. Accordingly, we consider the following two possibilities:

- Case I ("no consumers search" case): if $T > \frac{2\zeta(1-p)}{3} + \Delta$ (where $\zeta = \frac{\alpha+(1-\alpha)\mu_0}{\alpha+(1-\alpha)\mu_0+3(1-\alpha)(1-\mu_0)}$), then even the second max operator becomes zero, and therefore, both max operators vanish. That is, there are no consumers who search beyond the first firm. In this case, $\Gamma(\lambda^{tar}) = -2k \cdot \mu_0^2 \cdot \lambda^{tar} + p \cdot \mu_0 \cdot (\alpha+(1-\alpha)\mu_0) \cdot \big[1 - \frac{\lambda^{tar}}{4}\big]$, which is linearly decreasing in λ^{tar}. Therefore, a solution $\lambda^{tar} = \frac{4p(\alpha+(1-\alpha)\mu_0)}{p(\alpha+(1-\alpha)\mu_0)+8k\mu_0} \in (0,1)$ exists (and it is unique) if and only if $\Gamma(\lambda^{tar} = 1) < 0 \Leftrightarrow k > \overline{k}_1^{tar} := \frac{3p(\alpha+(1-\alpha)\mu_0)}{8\mu_0}$.

- Case II ("some consumers search" case): if $\frac{2\zeta(1-p)}{3} - \Delta < T < \frac{\zeta(1-p)}{3} + \Delta$, then the values of both max operators are between 0 and 1. Therefore, some consumers engage in search beyond the first firm. In this case, $\Gamma(\lambda^{tar}) = -2k \cdot \mu_0^2 \cdot \lambda^{tar} + p \cdot \mu_0 \cdot (\alpha+(1-\alpha)\mu_0) \cdot \big[1 - \frac{\lambda^{tar}}{4} - \frac{\lambda^{tar}}{6} \cdot \frac{1}{4\Delta} \cdot \big(\big(\frac{2(3-2\mu_0\lambda^{tar})}{3(2-\mu_0\lambda^{tar})} - \frac{2}{3}\big) \cdot \zeta(1-p) - (T-\Delta)\big)\big]$, so $\Gamma(\cdot)$ may no longer be monotonic. However, $\frac{\partial^2 \Gamma(\lambda^{tar})}{\partial \lambda^{tar 2}} = \frac{p(1-p) \cdot \mu_0^2 \cdot (\alpha+(1-\alpha)\mu_0)^2}{9\Delta(2-\mu_0\lambda^{tar})^3(\alpha+\mu_0(1-\alpha)+3(1-\mu_0)(1-\alpha))} > 0$. Because of this convexity, there can be only one $\lambda^{tar} \in (0,1)$ that makes $\Gamma(\lambda) = 0$ if and only if $\Gamma(1) < 0 \Leftrightarrow k > \frac{p(\alpha+(1-\alpha)\mu_0)}{2\mu_0} \cdot \big[\frac{3}{4} - \frac{1}{24\Delta}\big(\frac{2(1-\mu_0)}{3(2-\mu_0)} \cdot \zeta(1-p) - (T-\Delta)\big)\big]$, a sufficient condition for which is

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$$k > \overline{k}_2^{tar} := \tfrac{3p(\alpha+(1-\alpha)\mu_0)}{8\mu_0}.$$

Therefore, in both cases, there is a unique λ^{tar} if k is sufficiently large. ∎

Proof of Proposition 5

Under non-targeted advertising, the equilibrium indirect demand is obtained by plugging in $\widetilde{\sigma}_A = \lambda^{non} \cdot q_A$:

$$D_o^{Ind*} = \tfrac{\mu_0 q_A \lambda^{non}}{6} \Big(\tfrac{\lambda^{non} q_A}{2} \tfrac{\mathbb{E}^{non}[\max\{0, u_A - p\}|\theta^{1,1}, u_B = 0] - (T - \Delta)}{2\Delta} + (1 - \lambda^{non} q_A) \tfrac{\mathbb{E}^{non}[\max\{0, u_A - p\}|\theta^{0,1}, u_B = 0] - (T - \Delta)}{2\Delta} \Big).$$

The equilibrium expected indirect demand under targeted advertising is similarly obtained by plugging in the equilibrium advertising strategy $\widetilde{\sigma}_A = \sigma^{tar*} = (\lambda^{tar} \cdot q_A, 0)$:

$$D^{Ind*} = \mu_0 q_A \big(\alpha + (1-\alpha)\mu_0\big) \tfrac{\lambda^{tar}}{6} \Big(\tfrac{\lambda^{tar} q_A}{2} \tfrac{\mathbb{E}[\max\{0, u_A - p\}|\theta^{1,1}, u_B = 0] - (T - \Delta)}{2\Delta} + (1 - \lambda^{tar} q_A) \tfrac{\mathbb{E}[\max\{0, u_A - p\}|\theta^{0,1}, u_B = 0] - (T - \Delta)}{2\Delta} \Big).$$

If $\alpha \to 1$, then $D^{Ind*}|_{\alpha=1} = \mu_0 \cdot q_A \cdot \tfrac{\lambda}{6} \cdot \big(\tfrac{\lambda \cdot q_A}{2} \cdot \tfrac{\mathbb{E}[\max\{0, u_A - p\}|\theta^{1,1}, u_B = 0] - (T - \Delta)}{2\Delta} + (1 - \lambda \cdot q_A) \cdot \tfrac{\mathbb{E}[\max\{0, u_A - p\}|\theta^{0,1}, u_B = 0] - (T - \Delta)}{2\Delta} \big)$, which is greater than the indirect demand for targeted advertising. This is because $\lambda|_{\alpha=1} > \lambda^{non}$ and $\mathbb{E}[\max\{0, u_A - p\}|\theta, u_B = 0]|_{\alpha=1} \geq \mathbb{E}^{non}[\max\{0, u_A - p\}|\theta, u_B = 0]$. On the other hand, if $\alpha \to 0$, then $D^{Ind*}|_{\alpha=0} = \mu_0^2 \cdot q_A \cdot \tfrac{\lambda}{6} \cdot \big(\tfrac{\lambda \cdot q_A}{2} \cdot \tfrac{\mathbb{E}[\max\{0, u_A - p\}|\theta^{1,1}, u_B = 0] - (T - \Delta)}{2\Delta} + (1 - \lambda \cdot q_A) \cdot \tfrac{\mathbb{E}[\max\{0, u_A - p\}|\theta^{0,1}, u_B = 0] - (T - \Delta)}{2\Delta} \big)$, which is smaller than the indirect demand for targeted advertising. This is because $\lambda|_{\alpha=0} < \lambda^{non}$ and $\mathbb{E}[\max\{0, u_A - p\}|\theta, u_B = 0]|_{\alpha=0} = \mathbb{E}^{non}[\max\{0, u_A - p\}|\theta, u_B = 0]$. ∎

Proof of Proposition 6

There are two different λ^{tar} depending on the case in the proof of Proposition 4. Case I (no consumers search) where T is sufficiently large, i.e., $T > \tfrac{2\zeta(1-p)}{3} + \Delta$, is straightforward because the closed-form expression of $\lambda^{tar} = \tfrac{4p(\alpha+(1-\alpha)\mu_0)}{p(\alpha+(1-\alpha)\mu_0)+8k\mu_0}$ is simple: $\tfrac{\partial \lambda^{tar}}{\partial k} = -\tfrac{32p\mu_0(\alpha+(1-\alpha)\mu_0)p}{\big(p(\alpha+(1-\alpha)\mu_0)+8k\mu_0\big)^2} \leq 0$, and $\tfrac{\partial \lambda^{tar}}{\partial p} = -\tfrac{32k \cdot \mu_0(\alpha+(1-\alpha)\mu_0)}{\big(p(\alpha+(1-\alpha)\mu_0)+8k\mu_0\big)^2} \geq 0$. The remaining proof of results for Case II (some consumers search) is presented here, i.e., $\tfrac{2\zeta(1-p)}{3} - \Delta < T < \tfrac{\zeta(1-p)}{3} + \Delta$ is satisfied. This implies that $T < 3\Delta$, which will be used later. Recall that λ^{tar} solves $\Gamma(\lambda) = 0$, where $\Gamma(\lambda)$ is defined in equation (19). In this case, $\Gamma(\lambda) = -2k\mu_0^2 \cdot \lambda + p \cdot \mu_0(\alpha + (1-\alpha)\mu_0) \cdot \big[1 - \tfrac{\lambda}{4} - \tfrac{\lambda}{6} \cdot \tfrac{1}{4\Delta}\big(\tfrac{2(1-\mu_0\lambda)}{3(2-\mu_0\lambda)} \cdot \zeta(1-p) - (T-\Delta)\big)\big]$.

1. First, $\tfrac{\partial \Gamma(\lambda)}{\partial k} = 0$ must hold. Therefore, $-2\mu_0 \cdot \lambda = \tfrac{\partial \lambda}{\partial k} \cdot \xi$, where

$$\xi = 2k\mu_0 + p \cdot \big(\alpha + (1-\alpha)\mu_0\big) \cdot \Big[\tfrac{1}{4} + \tfrac{1}{24\Delta} \cdot \big(\tfrac{2\big((2-\mu_0\lambda)^2 - 2\big)}{3(2-\mu_0\lambda)^2} \cdot \zeta(1-p) - (T-\Delta)\big)\Big].$$

The left-hand side of $-2\mu_0 \cdot \lambda = \tfrac{\partial \lambda}{\partial k} \cdot \xi$ is negative. We will show that $\xi \geq 0$, which proves that $\tfrac{\partial \lambda}{\partial k} \leq 0$. Note that $-1 < \tfrac{(2-\mu_0\lambda)^2 - 2}{(2-\mu_0\lambda)^2} < \tfrac{1}{2}$ and $3(T - \Delta) < \zeta(1-p) < \tfrac{3}{2}(T + \Delta)$. Therefore, $-(T + \Delta) < \tfrac{2\big((2-\mu_0\lambda)^2 - 2\big)}{3(2-\mu_0\lambda)^2} \cdot \zeta(1-p) < \tfrac{T+\Delta}{2}$. Therefore, the constant multiplied by $\tfrac{\partial \lambda}{\partial k}$, ξ, is greater than

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$2k\mu_0^2 + p \cdot \mu_0\big(\alpha + (1-\alpha)\mu_0\big) \cdot \big[\frac{1}{4} - \frac{T}{12\Delta}\big]$. Then, $T < 3\Delta$ implies that the expression in brackets is nonnegative, and hence, so is $\xi \geq 0$. This proves $\frac{\partial \lambda}{\partial k} \leq 0$.

2. Second, $\frac{\partial \Gamma(\lambda)}{\partial T} = 0$ must hold, i.e., $p \cdot \mu_0(\alpha + (1-\alpha)\mu_0) \cdot \frac{\lambda}{24\Delta} = \frac{\partial \lambda}{\partial T} \cdot \xi$. The left-hand side is positive, and the right-hand side is $\frac{\partial \lambda}{\partial T}$ multiplied by $\xi \geq 0$ from the analysis of $\frac{\partial \lambda}{\partial T}$. Therefore, $\frac{\partial \lambda}{\partial T} \geq 0$.

3. Third, differentiating $\Gamma(\lambda) = 0$ with respect to p, we obtain $p\mu_0(1-\mu_0)\Psi_p(\lambda) = \frac{\partial \lambda}{\partial p} \cdot \xi$, where $\Psi_p(\lambda) := 1 - \frac{\lambda}{4} - \frac{\lambda}{24\Delta}\big(\frac{2(1-\mu_0\lambda)}{3(2-\mu_0\lambda)} \cdot \zeta(1-p) - (T-\Delta)\big)$. Because $\Gamma(\lambda) = 0$, we can rewrite the expression $\Psi_p(\lambda) = \big(\frac{2k\mu_0}{p(\alpha+(1-\alpha)\mu)} - \frac{1}{24\Delta}\big)\lambda$, which is positive for all values of $p \in [0,1]$ if $k > \frac{\alpha+(1-\alpha)\mu_0}{48\mu_0 \cdot \Delta}$. Therefore, if k is sufficiently large, then $\Psi_p(\lambda) \geq 0$, and hence, $\frac{\partial \lambda}{\partial p} \geq 0$.

∎

Proof of Proposition 7

Similar to the proof of Proposition 6, there are two different cases. In Case I (no consumers search), T is sufficiently large, i.e., $T > \frac{2\zeta(1-p)}{3} + \Delta$ and $\lambda^{tar} = \frac{4p(\alpha+(1-\alpha)\mu_0)}{p(\alpha+(1-\alpha)\mu_0)+8k\mu_0}$. Thus, it is easy to see that $\frac{\partial \lambda^{tar}}{\partial \alpha} = \frac{32k \cdot \mu_0(1-\mu_0)p}{\big(p(\alpha+(1-\alpha)\mu_0)+8k\mu_0\big)^2} \geq 0$. In Case II (some consumers search), $\frac{2\zeta(1-p)}{3} - \Delta < T < \frac{\zeta(1-p)}{3} + \Delta$ holds. This implies that $T < 3\Delta$. Recall that λ^{tar} solves $\Gamma(\lambda) = 0$, where $\Gamma(\lambda)$ is defined in equation (19). In this case, $\Gamma(\lambda) = -2k\mu_0^2 \cdot \lambda + p \cdot \mu_0(\alpha + (1-\alpha)\mu_0) \cdot \big[1 - \frac{\lambda}{4} - \frac{\lambda}{6} \cdot \frac{1}{4\Delta}\big(\frac{2(1-\mu_0\lambda)}{3(2-\mu_0\lambda)} \cdot \zeta(1-p) - (T-\Delta)\big)\big]$. $\frac{\partial \Gamma(\lambda)}{\partial \alpha} = 0$ must hold, which can be expressed as $p \cdot \mu_0(1-\mu_0) \cdot \Psi_\alpha(\lambda) = \frac{\partial \lambda}{\partial \alpha} \cdot \xi$, where

$$\Psi_\alpha(\lambda) := 1 - \frac{\lambda}{4} - \frac{\lambda}{6} \cdot \frac{1}{4\Delta} \cdot \left(\frac{2(1-\mu_0\lambda)}{3(2-\mu_0\lambda)} \cdot \zeta(1-p) \cdot \frac{3+\alpha+(1-\alpha)\mu_0+3(1-\alpha)(1-\mu_0)}{\alpha+(1-\alpha)\mu_0+3(1-\alpha)(1-\mu_0)} - (T-\Delta)\right).$$

Dividing both sides of $\Gamma(\lambda) = 0$ $p \cdot \mu_0(\alpha + (1-\alpha)\mu_0)$ gives $\frac{2k \cdot \mu_0^2 \cdot \lambda}{p \cdot \mu_0 \cdot (\alpha+(1-\alpha)\mu_0)} = 1 - \frac{\lambda}{4} - \frac{\lambda}{6} \cdot \frac{1}{4\Delta}\big(\frac{2(1-\mu_0\lambda)}{3(2-\mu_0\lambda)} \cdot \zeta(1-p) - (T-\Delta)\big)$. Plugging this into $\Psi_\alpha(\lambda)$ shows that $\Psi_\alpha(\lambda) < 0$ if and only if $\lambda\big[\frac{2k \cdot \mu_0^2}{p \cdot \mu_0 \cdot (\alpha+(1-\alpha)\mu_0)} - \frac{1}{24\Delta} \cdot \frac{1-\mu_0\lambda}{2-\mu_0\lambda} \cdot \zeta(1-p) \cdot \frac{1}{\alpha+(1-\alpha)\mu_0+3(1-\alpha)(1-\mu_0)}\big] < 0$. As $\alpha \to 1$, $\Psi_\alpha(\lambda)|_{\alpha=1} = \lambda\big[\frac{2k \cdot \mu_0}{p} - \frac{1}{24\Delta} \cdot \frac{1-\mu_0\lambda}{2-\mu_0\lambda}\big]$. Note that $\frac{1-\mu_0}{2-\mu_0} < \frac{1-\mu_0\lambda}{2-\mu_0\lambda} < \frac{1}{2}$. Therefore, $\Psi_\alpha(\lambda)|_{\alpha=1} < 0$ if $\frac{2k \cdot \mu_0}{p} - \frac{1}{24\Delta} \cdot \frac{1-\mu_0}{2-\mu_0} < 0$, i.e., $\frac{k}{p} < \frac{1}{48\Delta} \cdot \frac{1-\mu_0}{\mu_0(2-\mu_0)}$. On the other hand, as $\alpha \to 0$, $\Psi_\alpha(\lambda) \to \lambda\big[\frac{2k}{p} - \frac{1}{24\Delta} \cdot \frac{1-\mu_0\lambda}{2-\mu_0\lambda} \cdot \frac{\mu_0}{(3-2\mu_0)^2}\big]$. Therefore, $\Psi_\alpha(\lambda)|_{\alpha=0} > 0$ if $\frac{k}{p} > \frac{1}{48\Delta} \cdot \frac{1-\mu_0\lambda}{2-\mu_0\lambda} \cdot \frac{\mu_0}{(3-2\mu_0)^2}$, whose sufficient condition is $\frac{k}{p} > \frac{1}{96\Delta} \cdot \frac{\mu_0}{(3-2\mu_0)^2}$. Hence, λ is nonmonotonic in α if $\frac{p}{96\Delta} \cdot \frac{\mu_0}{(3-2\mu_0)^2} < k < \frac{p}{48\Delta} \cdot \frac{1-\mu_0}{\mu_0(2-\mu_0)}$. ∎

Proof of Proposition 8

To compare λ^{non} and $\mu_0\lambda^{tar}$, we use the following proof strategy. In the proof of Proposition 3, we characterized the equilibrium level of non-targeted advertising λ^{non} which solves $\Gamma^{non}(\lambda) = 0$, where the decreasing function $\Gamma^{non}(\lambda)$ is defined in equation (16). Therefore, $\lambda^{non} \geq \mu_0\lambda^{tar}$ if and only if $\Gamma^{non}(\mu_0\lambda^{tar}) \geq 0$. Using $\Gamma(\lambda^{tar}) = 0$, this condition is equivalent to $\Gamma^{non}(\mu_0\lambda^{tar}) - \kappa \cdot \Gamma(\lambda^{tar}) \geq 0$ for any constant κ.

Note that $\Gamma^{non}(\mu_0\lambda^{tar}) = -2k\mu_0\lambda^{tar} + p\mu_0\big[1 - \frac{\mu_0\lambda^{tar}}{4} - \frac{\mu_0\lambda^{tar}}{6}\big(X^{non} - \frac{Y^{non}}{2}\big)\big]$, where $X^{non} = \max\{0, \frac{1}{2\Delta}\big(\frac{\mu_0}{3-2\mu_0} \frac{3-2\mu_0\lambda}{3(2-\mu_0\lambda)}(1-p) - (T-\Delta)\big)\}$ and $Y^{non} = \max\{0, \frac{1}{2\Delta}\big(\frac{\mu_0}{3-2\mu_0} \frac{2}{3}(1-p) - (T-\Delta)\big)\}$. Also, λ^{tar} satisfies because $\Gamma(\lambda^{tar}) = 0 =$

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$-2k\mu_0^2\lambda^{tar} + p\mu_0(\alpha + (1-\alpha)\mu_0)\big[1 - \frac{\lambda^{tar}}{4} - \frac{\lambda^{tar}}{6}\big(X^{tar} - \frac{Y^{tar}}{2}\big)\big]$, where $X^{tar} = \max\big\{0, \frac{1}{2\Delta}\big(\frac{3-2\mu_0\lambda}{3(2-\mu_0\lambda)}\zeta(1-p) - (T-\Delta)\big)\big\}$ and $Y^{tar} = \max\big\{0, \frac{1}{2\Delta}\big(\frac{2\zeta(1-p)}{3} - (T-\Delta)\big)\big\}$. We use the following properties in the rest of the proof:

1. Because of the first-order condition for the targeting case, $\frac{\lambda^{tar}}{4} + \frac{\lambda^{tar}}{6}\big(X^{tar} - \frac{Y^{tar}}{2}\big) = 1 - \frac{2k\mu_0\lambda^{tar}}{p(\alpha+(1-\alpha)\mu_0)}$.

2. Because of the first-order condition for the non-targeting case, $\frac{\lambda^{non}}{4} + \frac{\lambda^{non}}{6}\big(X^{non} - \frac{Y^{non}}{2}\big) = 1 - \frac{2k\lambda^{non}}{p\mu_0}$.

3. As $\alpha \to 0$, $X^{tar} \to_{\alpha\to 0} X^{non}$ and $Y^{tar} \to_{\alpha\to 0} Y^{non}$. Similarly, as $\alpha \to 1$, $\zeta \to 1$.

1. First, we show that as $\alpha \to 0$, $\mu_0\lambda^{tar} < \lambda^{non}$ by proving that $\Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar})/\mu_0 \geq 0$. Using property #3, $\Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar})/\mu_0 = p\cdot\mu_0\big[\frac{(1-\mu_0)\lambda^{tar}}{4} + \frac{(1-\mu_0)\lambda^{tar}}{6}\cdot\frac{1}{4\Delta}\big(X^{tar} - \frac{Y^{tar}}{2}\big)\big] = p\mu_0(1-\mu_0)\big(1 - \frac{2k\mu_0\lambda^{tar}}{p(\alpha+(1-\alpha)\mu_0)}\big) > 0$. A sufficient condition for $\lambda^{tar} = 1$ is $k < \frac{p}{2\mu_0}$. By continuity, this proves that $\lambda^{non} \geq \mu_0\lambda^{tar}$ for α close to 0.

2. Second, as $\alpha \to 1$, we identify a sufficient condition for $\mu_0\lambda^{tar} \leq \lambda^{non}$. Equivalently, we identify a sufficient condition for $0 \leq \Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar}) = -2k\mu_0(1-\mu_0)\lambda^{tar} + \mu_0\cdot p\big(\frac{(1-\mu_0)\lambda^{tar}}{4} + \frac{(1-\mu_0)\lambda^{tar}}{6}(X^{tar} - \frac{Y^{tar}}{2}) - \frac{\mu_0\lambda^{tar}}{6}(X^{non} - \frac{Y^{non}}{2}) + \frac{\mu_0\lambda^{tar}}{6}(X^{tar} - \frac{Y^{tar}}{2})\big)$. We focus on a region of consumer search cost T which satisfies the following properties: (1) Search cost T is sufficiently small such that as $\alpha \to 1$, $X^{tar}, Y^{tar} \to 1$, i.e., all consumers who are dissatisfied with the first firm search for the second firm. Therefore, $X^{tar} - \frac{Y^{tar}}{2} = \frac{1}{2}$. (2) Under non-targeted advertising, $X^{non} = 0$ and $Y^{non} = \frac{1}{2\Delta}\big(\frac{\mu_0}{3-2\mu_0}\cdot\frac{2}{3}(1-p) - (T-\Delta)\big) \in (0,1)$, in which case $X^{non} - \frac{Y^{non}}{2} < 0$. Using these properties, together with Property 1 above, $\Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar}) \geq -2k\mu_0(1-\mu_0)\lambda^{tar} + \mu_0 p\big((1-\mu_0)(1 - \frac{2k\mu_0\lambda^{tar}}{p}) + \frac{\mu_0\lambda^{tar}}{12}\big)$. It is decreasing in λ^{tar} if $-2k\mu_0(1-\mu_0) - 2k\mu_0^2(1-\mu_0) + \frac{p\mu_0^2}{12} < 0 \Leftrightarrow k > \frac{p\mu_0}{24(1-\mu_0)(1+\mu_0)}$. A sufficient condition for $\Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar}) \geq 0$ is obtained by plugging in $\lambda^{tar} = 1$, i.e., $-2k\mu_0(1-\mu_0) + \mu_0 p\big((1-\mu_0)(1 - \frac{2k\mu_0}{p}) + \frac{\mu_0}{12}\big) \geq 0$ if and only if $k \leq \frac{12-11\mu_0}{2(1-\mu_0)(1+\mu_0)}p$. For the existence of $\lambda^{tar} \in (0,1)$, it must be $k > \frac{p}{2\mu_0}[\frac{3}{4} - \frac{1}{12}] = \frac{3p}{8\mu_0}$. Therefore, for $\Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar}) \geq 0$ to hold at a non-empty parameter region, $\frac{3p}{8\mu_0} \leq \frac{(12-11\mu_0)p}{2(1-\mu_0)(1+\mu_0)}$, which holds if $\mu_0 \geq \frac{24-\sqrt{453}}{41} \approx 0.066$. This provides a sufficient condition for $\Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar}) \geq 0$ given $\alpha \to 1$ and T relatively small, thus proving $\mu_0\lambda^{tar} \leq \lambda^{non}$.

3. Third, as $\alpha \to 1$, suppose T is sufficiently large that no consumer search beyond the first firm. This corresponds to Case I where $X^{tar}, Y^{tar}, X^{non}, Y^{non} \to 0$. Then, the same result cannot hold, i.e., $0 > \Gamma^{non}(\mu_0\lambda^{tar}) - \Gamma(\lambda^{tar}) = -2k\mu_0(1-\mu_0)(1+\mu_0)\cdot\frac{4p}{p+8k\mu_0} + \mu_0(1-\mu_0)p$, which holds if and only if $k > \frac{p}{8}$. This condition is already assumed for the existence of $\lambda^{tar} \in (0,1)$. This proves that $\mu_0\lambda^{tar} \geq \lambda^{non}$ for $\alpha \to 1$ if T is large such that there is no or very little consumer search. $\blacksquare$

Proof of Corollary 1

We compare the profits in Case I (no consumers search) for both non-targeted and targeted advertising where the average consumer search cost T is sufficiently large so that no consumer searches beyond the first firm. In this case, the profit functions simplify significantly, and we obtain $\mathbb{E}\Pi^{non*}(q) - \mathbb{E}\Pi^{tar*}(q) =$

$$\frac{16kp^2q^2(1-\mu_0)\mu_0^2\big(p^2(1+\mu_0)(\alpha+(1-\alpha)\mu_0)^2 - 16k\mu_0 p(1-\alpha)(\alpha+(1-\alpha)\mu_0) - 64k^2\alpha(\alpha(1-\mu_0)+2\mu_0)\big)}{(8k+p\mu_0)^2\big(8k\mu_0 + p(\alpha+(1-\alpha)\mu_0)\big)^2}.$$

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It is straightforward to show that it is monotonically decreasing in α, i.e., $\frac{\partial}{\partial \alpha}(\mathbb{E}\Pi^{non*}(q) - \mathbb{E}\Pi^{tar*}(q)) = \frac{256k^2p^2q^2\mu_0^3(1-\mu_0)(\alpha+(1-\alpha)\mu_0)}{8k\mu_0+p(\alpha+(1-\alpha)\mu_0)} < 0$. Then, $\mathbb{E}\Pi^{non*}(q) - \mathbb{E}\Pi^{tar*}(q) < 0$ if and only if $\alpha > \widehat{\alpha} = \frac{p\cdot\mu_0}{8k-p(1-\mu_0)}$. $\blacksquare$

Proof of Proposition 9

Firm A's direct demand, given the firm's chosen level of advertisement $\widetilde{\sigma}_j^g$ and $\widetilde{\sigma}_j^b$, is $D_A^{Dir}(p_A, \widetilde{\sigma}_A,; p^*, \sigma^*) = \frac{\mu_0}{2} \cdot \left[(\alpha + (1-\alpha)\mu_0) \cdot \widetilde{\sigma}_A^g \cdot (1 - \frac{\sigma_B^{g*}}{2}) + (1-\alpha)(1-\mu_0) \cdot \widetilde{\sigma}_A^b \cdot (1 - \frac{\sigma_B^{b*}}{2})\right] \cdot (1 - p_A)$. The consumer's search decision when she is not satisfied with the first firm's product (i.e., $u_{iB} = 0$) under endogenous pricing is as follows: some consumers may visit firm B first and subsequently decide whether to search for firm A. If $u_{iB} = 0$, the consumer does not buy the product, and she searches for the other firm B if $\Pr[m_i = 1|\theta, u_{iB} = 0] \cdot \frac{1}{2} \max\{0, \phi_i - p_A^*\} - t \geq 0 \Leftrightarrow \phi_i \geq p_A^* + \frac{2t}{\Pr[m_i=1|\theta,u_{iB}=0]}$. On the other hand, if $u_{iB} = 1$, the consumer buys the product without searching for firm B if $1 - p_B^* \geq \frac{1}{2}(1 - p_A^*) - t \Leftrightarrow p_A^* \geq 2p_B^* - 1 - 2t$. In a symmetric equilibrium with $p_A^* = p_B^*$, this condition does not hold, and therefore, consumers who are satisfied with the first firm do not search for the second firm. Therefore, firm A's indirect demand from those who visit firm B first and subsequently search for firm A is $D_A^{Ind}(p_A; p^*, \sigma^*) = \frac{\mu_0}{2} \cdot \frac{1}{2} \cdot \left[(\alpha + (1-\alpha)\mu_0) \cdot \left((1 - \widetilde{\sigma}_A^g)\sigma_B^{g*} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{0,1},u_{iB}=0]}) + \frac{\widetilde{\sigma}_A^g \cdot \sigma_B^{g*}}{2} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{1,1},u_{iB}=0]})\right) + (1-\alpha)(1-\mu_0) \cdot \left((1 - \widetilde{\sigma}_A^b)\sigma_B^{b*} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{0,1},u_{iB}=0]}) + \frac{\widetilde{\sigma}_A^b \cdot \sigma_B^{b*}}{2} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{1,1},u_{iB}=0]})\right)\right]$ if $p_A < p_A^* + \frac{2t}{\Pr[m_i=1|\theta,u_{iB}=0]}$, which must hold in equilibrium, where $p_A = p_A^*$. However, if $p_A \geq p_A^* + \frac{2t}{\Pr[m_i=1|\theta^{1,1},u_{iB}=0]}$, then $D_A^{Ind}(p_A; p^*, \sigma^*) = \frac{\mu_0}{2} \cdot \frac{1}{2} \cdot \left[(\alpha + (1-\alpha)\mu_0) \cdot \left((1 - \widetilde{\sigma}_A^g)\sigma_B^{g*} + \frac{\widetilde{\sigma}_A^g \cdot \sigma_B^{g*}}{2}\right) + (1-\alpha)(1-\mu_0) \cdot \left((1 - \widetilde{\sigma}_A^b)\sigma_B^{b*} + \frac{\widetilde{\sigma}_A^b \cdot \sigma_B^{b*}}{2}\right)\right] \cdot (1 - p_A)$.

Given these demands, the firm's expected profit is $p_A \cdot \left(D_A^{Dir}(p_A; p^*, \sigma^*) + D_A^{Ind}(p_A; p^*, \sigma^*)\right) - k \cdot \left(\mu_0 \cdot \widetilde{\sigma}_A^g + (1-\mu_0) \cdot \widetilde{\sigma}_A^b\right)^2$. The optimal price in a symmetric equilibrium must satisfy the first-order condition $\frac{\partial \pi_A(p_A; p^*, \sigma^*)}{\partial p_A}|_{p_A=p_A^*} = 0 \Leftrightarrow D_A^{Dir}(p^*; p^*, \sigma^{g*}) + D_A^{Ind}(p^*; p^*, \sigma^{g*}) + p^* \cdot \frac{\partial D_A^{Dir}(p^*; p^*, \sigma^{g*})}{\partial p} = 0$, i.e.,

$$\sigma^{g*} \cdot (1 - \frac{\sigma^{g*}}{2})\left(1 - p^* + \frac{1}{2}(1 - 2p^* - \frac{2t}{\Pr(m_i|\theta, u_{iB} = 0)})\right) = 0. \tag{19}$$

Therefore, $p^* = \frac{1}{5}\left(3 - \frac{2t}{\Pr(m_i|\theta,u_{iB}=0)}\right)$.

To ensure that at least some consumers engage in searching beyond the first firm in this equilibrium, we need to check a condition under which a consumer of type $\phi_i = 1$ will search if the consumer is dissatisfied with the first firm, i.e., $\Pr[m_i = 1|\theta, u_{iB} = 0] \cdot \frac{1}{2} \max\{0, 1 - p_A^*\} - t_i \geq 0$. Equivalently, $1 \geq p_A^* + \frac{2t}{\Pr[m_i=1|\theta,u_{iB}=0]} = \frac{1}{5}\left(3 - \frac{2t}{\Pr(m_i|\theta,u_{iB}=0)}\right) + \frac{2t}{\Pr[m_i=1|\theta,u_{iB}=0]} = \frac{3}{5} + \frac{8t}{5\Pr(m_i|\theta,u_{iB}=0)}$, i.e., $t < \frac{\alpha+(1-\alpha)\mu_0}{4\left(\alpha+(1-\alpha)\mu_0+3(1-\alpha)(1-\mu_0)\right)}$.

Next, we identify firms' optimal advertising strategy. The first-order condition is $\frac{\partial \pi_A(\widetilde{\sigma}_A^g; p^*, \sigma^*)}{\partial \widetilde{\sigma}_A^g}|_{\widetilde{\sigma}_A^g=\sigma^{g*}} = 0$

$$\Longleftrightarrow \frac{\mu_0(\alpha + (1-\alpha)\mu_0)}{2} \cdot p^*\left[(1 - p^*)(1 - \frac{\sigma^{g*}}{2}) - \frac{\sigma^{g*}}{4} \cdot (1 - p^* - \frac{2t}{\Pr[m_i = 1|\theta^{0,1}, u_{iB} = 0]})\right] - 2k \cdot \mu_0^2 \cdot \sigma^{g*} = 0. \tag{20}$$

This equation is linear in σ^{g*}. We invoke the intermediate value theorem for the existence of $\sigma^{g*} \in (0,1)$. If $\sigma^{g*} = 0$, then the left-hand side is positive, so the equation cannot hold. If $\sigma^{g*} = 1$, then the left-hand side is

$\frac{\mu_0(\alpha+(1-\alpha)\mu_0)}{2} \cdot p^* \left[\frac{1-p^*}{4} + \frac{t}{2\Pr[m_i=1|\theta^{0,1},u_{iB}=0]}\right] - 2k \cdot \mu_0^2$, which is negative if and only $k > \frac{\alpha+(1-\alpha)\mu_0}{4\mu_0} \cdot p^* \left[\frac{1-p^*}{4} + \frac{t}{2\Pr[m_i=1|\theta^{0,1},u_{iB}=0]}\right]$. The equilibrium advertising level is $\sigma^{g*} = \frac{(\alpha+(1-\alpha)\mu_0)p^*(1-p^*)}{(\alpha+(1-\alpha)\mu_0)p^*\left(\frac{3(1-p^*)}{4} - \frac{t}{2\Pr(m_i=1|\theta,u_{iB}=0)}\right)+2k\mu_0}$.

Last, a sufficient condition for $\sigma^b = 0$ to be optimal is identified. First, $\frac{\partial \pi_A(\tilde{\sigma}_A^b;p^*,\sigma^*)}{\partial \tilde{\sigma}_A^b} \leq 0$, i.e., $p^* \cdot \frac{\mu_0(1-\alpha)(1-\mu_0)}{2}\left((1-p^*)(1-\frac{\sigma^{b*}}{2}) - \frac{\sigma^{b*}}{4}(1-p^* - \frac{2t}{\Pr(m_i=1|\theta,u_{iB}=0)})\right) - 2k(1-\mu_0)\left(\mu_0\tilde{\sigma}_A^g + (1-\mu_0)\tilde{\sigma}_A^b\right) \leq 0$. After plugging in $\sigma^{b*} = 0$ and $\tilde{\sigma}_A^b = 0$, we obtain $p^*(1-p^*) \cdot \frac{\mu_0(1-\alpha)(1-\mu_0)}{2} - 2k(1-\mu_0)\mu_0\sigma^{g*} \leq 0$, i.e., $k \geq \frac{p^*(1-p^*)(1-\alpha)}{4\sigma^{g*}} = \frac{1-\alpha}{4(\alpha+(1-\alpha)\mu_0)} \cdot \left[(\alpha+(1-\alpha)\mu_0)p^*\left(\frac{3(1-p^*)}{4} - \frac{t}{2\Pr(m_i=1|\theta,u_{iB}=0)}\right) + 2k\mu_0\right]$. Therefore, $\left(1 - \frac{(1-\alpha)\mu_0}{2(\alpha+(1-\alpha)\mu_0)}\right)k \geq \frac{(1-\alpha)p^*}{4} \cdot \left[\frac{3(1-p^*)}{4} - \frac{t}{2\Pr(m_i=1|\theta,u_{iB}=0)}\right]$, or $k \geq \frac{(1-\alpha)(\alpha+(1-\alpha)\mu_0)p^*}{4(2\alpha+(1-\alpha)\mu_0)} \cdot \left[\frac{3(1-p^*)}{4} - \frac{t}{2\Pr(m_i=1|\theta,u_{iB}=0)}\right]$. ∎

Proof of Lemma 3

The indirect demand in equilibrium is $D_A^{Ind}(p,\sigma^*) = \frac{\mu_0(\alpha+(1-\alpha)\mu_0)}{4}(1-\frac{\sigma^{g*}}{2})\sigma^{g*} \cdot (1 - p - \frac{2t}{\Pr(m_i=1|\theta,u_{iB}=0)})$, and therefore, clearly, $\frac{\partial D_A^{Ind}(p,\sigma^*)}{\partial p} < 0$. It is also straightforward to show that $\frac{\partial p^*}{\partial \alpha} > 0$ by directly differentiating $p^* = \frac{3}{5} - \frac{2t}{5\Pr(m_i=1|\theta,u_{iB}=0)}$, where $\Pr(m_i=1|\theta,u_{iB}=0) = \frac{\alpha+(1-\alpha)\mu_0}{\alpha+(1-\alpha)\mu_0+3(1-\alpha)(1-\mu_0)}$. ∎

Proof of Proposition 10

First, substituting p^* into σ^{g*} and then differentiating σ^{g*} with respect to α yields $\frac{d\sigma^{g*}}{d\alpha} =$

$$-\frac{20(1-\mu_0)\left(12t^3 \cdot W^2 - 36t^2 \cdot \mu_0 \cdot W + 27t \cdot W^2 - k \cdot [160\alpha\mu_0(1-\mu_0)t^2 \cdot \xi + 80\mu_0(3+2\mu_0)t^2 \cdot \xi + 120\mu_0 \cdot V^2 - 80\mu_0 t \cdot V^2]\right)}{\left(4t^2 \cdot W^2 + V \cdot (9V + 200k\mu_0) + 12t(2\alpha^2(1-\mu_0)^2 - \mu_0(3-2\mu_0) - \alpha(3-7\mu_0+4\mu_0^2))\right)^2},$$

where $W = \alpha + (1-\alpha)\mu_0 + 3(1-\alpha)(1-\mu_0)$ and $V = \alpha + (1-\alpha)\mu_0$. Therefore, if k is sufficiently large, the numerator of the expression is negative, which makes $\frac{\partial \sigma^{g*}}{\partial \alpha} > 0$.

Second, $\frac{\partial \pi_A(\sigma^{g*},p^*)}{\partial \alpha} = \frac{\partial p^*}{\partial \alpha} \cdot (D_A^{Dir}(\sigma^{g*},p^*) + D_A^{Ind}(\sigma^{g*},p^*)) + p^* \cdot \left(\frac{dD_A^{Dir}(\sigma^{g*},p^*)}{d\alpha} + \frac{dD_A^{Ind}(\sigma^{g*},p^*)}{d\alpha}\right) - \frac{dc(\sigma^{g*})}{d\alpha}$. $D_A^{Dir}(\sigma^{g*},p^*) = \frac{\mu_0(\alpha+(1-\alpha)\mu_0)}{2}\sigma^{g*}\left(1-\frac{\sigma^{g*}}{2}\right)(1-p^*)$, $D_A^{Ind}(p_A;p^*,\sigma^*) = \frac{\mu_0\left(\alpha+(1-\alpha)\mu_0\right)}{4} \cdot (1-\frac{\sigma^{g*}}{2})\sigma^{g*} \cdot (1 - p^* - \frac{2t}{\Pr[m_i=1|\theta^{0,1},u_{iB}=0]})$, and $c(\sigma^{g*}) = k \cdot (\mu_0\sigma^{g*})^2$. Using equation (19), $\frac{d\pi_A(\sigma^{g*},p^*)}{d\alpha} = \frac{d}{d\alpha}\left(\frac{\mu_0(\alpha+(1-\alpha)\mu_0)}{2} \cdot \sigma^{g*}(1-\frac{\sigma^{g*}}{2}) \cdot p^*\right) - 2k\mu_0^2\sigma^{g*} \cdot \frac{\partial \sigma^{g*}}{\partial \alpha}$. If t is sufficiently large so that no consumers search in equilibrium, then the profit function simplifies and indirect demand vanishes. Therefore, $\pi_A(\sigma^{g*},p^*) = \frac{\mu_0(\alpha+(1-\alpha)\mu_0)}{2} \cdot \sigma^{g*}(1-\frac{\sigma^{g*}}{2}) \cdot p^*(1-p^*) - k(\mu_0\sigma^{g*})^2$. In this case, $p^* = \frac{1}{2}$, and $g^* = \frac{2(\alpha+(1-\alpha)\mu_0)}{\alpha+(1-\alpha)\mu_0+32k\mu_0}$, and $\frac{\partial g^*}{\partial \alpha} = \frac{64k\mu_0(1-\mu_0)}{\left(\alpha+(1-\alpha)\mu_0+32k\mu_0\right)^2}$. After plugging these expressions into the profit function, we obtain $\frac{d\pi(\sigma^{g*},p^*)}{d\alpha} = \frac{256k^2(\alpha+(1-\alpha)\mu_0)(1-\mu_0)\mu_0^3}{\left(\alpha+(1-\alpha)\mu_0+32k\mu_0\right)^3} > 0$. ∎

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