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Structural-temporal approach to modeling of fracture dynamics in brittle media

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ABSTRACT: The latest results connected with application of the incubation time approach to problems of dynamic fracture of brittle materials including rocks and concretes are summarized. The incubation time based fracture criteria for intact media and media with cracks are discussed. Available experimental data on high rate fracture of different rock materials and incubation time based fracture criteria are used in order to evaluate critical parameters causing fracture in these materials. A unified interpretation of rate effects based on the structural-temporal approach is presented. It is shown that the temporal dependence of the dynamic spall strength and split tensile strength can be predicted by the incubation time criterion. It is shown that in spite of the fact that static strength of one material is smaller than that of another one, its dynamic strength measured in terms of the incubation time can be essentially higher. By comparing static and dynamic strength, it is possible to optimize (minimize) the energy input needed for fracture that is principally important in connection to industrial rock fracture processes.

1 INTRODUCTION

Understanding mechanisms underlying dynamic fracture of brittle media is one of the central challenges in modern rock mechanics. Dynamic loads working for fracture or fragmentation of rocks represent the essence of many industrial processes in mining and further handling of rock materials. Though for several decades it is known and generally recognized that the static fracture criteria (critical stress criterion for fracture of intact media and the Irwin's critical stress intensity factor criterion for fracture of cracked bodies) are not applicable to study fracture caused by dynamic loads, no conventional approach to the problem is formed to the moment.

Some years ago, a new incubation time based approach to predict all the variety of experimentally observed effects typical of dynamic fracture was proposed (Petrov & Utkin 1989, Petrov 1991, Petrov & Morozov 1994). It was shown that staying within the framework of linear elastic fracture mechanics it is possible to predict all the features typical of fracture caused by high rate loads (Petrov & Morozov 1994, Morozov & Petrov 2000, Petrov et al. 2003). And even more attractive is the fact, that the same critical fracture condition can be used for all load rates – from quasistatic situations, when the incubation time criterion repeats the classical fracture criteria, to extreme dynamic conditions, when the incubation time criterion is in a very good qualitative and quantitative agreement with experimentally observed phenomena.

In this paper recent progress on application of the general incubation time approach to problems of dynamic fracture of rock materials and concretes is presented.

2 INCUBATION TIME APPROACH

Experiments on the dynamic loading of solids reveal a number of effects indicating a fundamental difference between the fast dynamic rupture (breakdown) of materials and a similar process under slow quasistatic loads. For example, one of the basic problems in testing of dynamic strength properties of materials is associated with the dependence of the limiting rupture characteristics on the duration, amplitude, and growth rate of an external load, as well as on a number of other factors. While a critical value for strength parameter is a constant for a material in the static case, experimentally determined critical characteristics in dynamics are found to be strongly unstable, having a behavior that is unpredictable. The indicated (and some other) features of the behavior of materials subjected to pulsed loads are common for a number of seemingly quite different physical processes, such as dynamic fracture (crack initiation, propagation, arrest and spalling), cavitation in liquids, electrical breakdown in insulators, initiation of detonation in gaseous media, etc. Unified interpretation for fracture of solids, yielding and phase transforms is possible, constituting structural-time approach, based on the concept of the incubation time of a transient dynamic process.

The main difficulty in modeling the aforementioned effects of mechanical strength, yielding and phase transitions is the absence of an adequate limiting condition that determines the possibility of rupture, yield or phase transform. The problem can be solved by using both the structural fracture macromechanics and the concept of the incubation time of the corresponding process, representing the nature of kinetic processes underlying formation of macroscopic breaks, yield flow or phase transformation. The above effects become essential for impacts with periods comparable to the scale determined by the fracture incubation time that is associated with preparatory relaxation processes accompanying development of micro defects in the material structure.

The criterion of fracture based on a concept of incubation time makes it possible to predict unstable behavior of dynamic-strength characteristics. These effects are observed in experiments on the dynamic fracture of solids. The fracture criterion can be generalized (Petrov 2004):

$$\frac{1}{\tau}\cdot\int_{t-\tau}^{t}\left(\frac{F(t')}{F_c}\right)^{\alpha}dt' \le 1. \qquad (1)$$

Here, $F(t)$ is the intensity of a local force field causing the fracture (or structural transformation) of the medium, F_c is the static limit of the local force field, and τ is the incubation time associated with the dynamics of a relaxation process preparing the break. It actually characterizes *the strain (stress) rate sensitivity* of a material. The fracture time t_* is defined as the time at which condition Eq. (1) becomes equality. The parameter α characterizes *the sensitivity of a material to the intensity (amplitude)* of the force field causing fracture (or structural transformation).

Using an example of mechanical break of a material, one of the possible methods of interpreting and determining the parameter τ is proposed. It is assumed that a standard sample made of a material in question is subjected to tension and is broken into two parts under a stress P arising at a certain time $t = 0$: $F(t) = PH(t)$, where $H(t)$ is the Heaviside step function. In the case of quasi-brittle fracture, the material should unload, and the local stress at the break point should decreases rapidly (but not instantaneously) from P to 0. In this case, the corresponding unloading wave is generated, propagates over the sample, and can be detected by well-known (e.g., interferometric) methods. The stress variation at the break point can be conditionally represented by the dependence $\sigma(t) = P - Pf(t)$, where $f(t)$ varies from 0 to 1 within a certain time interval T. The case $f(t) = H(t)$ corresponds to the classical theory of strength. In other words, according to the classical approach, break occurs instantaneously ($T = 0$). In practice, the break of a material (sample) is a process in time, and the function $f(t)$ describes the *micro- scale level* kinetics of the transition from a conditionally defect-free state ($f(0) = 0$) to the completely broken state at the given point ($f(t_*) = 1$) that can be associated with the macro-fracture event (Figure 1). On the other hand, applying the fracture criterion (1) to *macro- scale level* situation ($F(t) = PH(t)$), the relation for time to fracture $t_* = T = \tau$ for $P = F_c$ is received.

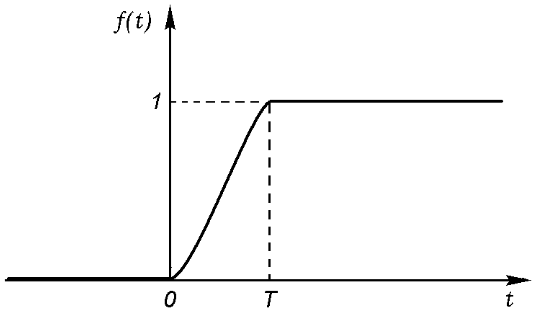

Figure 1. Schematic representation of *micro- scale level* kinetics of fracture of a sample at the break point.

In other words, the incubation time introduced above is equal to the duration of the fracture process after the stress in the material has reached the static breaking strength *on the given scale level*. This duration can be measured experimentally statically fracturing samples and controlling the rupture process by different possible methods, e.g., measuring the time of the increase pressure at the unloading wave front, which can be determined by the interferometric (visar-based, or photoelasticity-based) method using the velocity profile of points of the sample boundary. Below, we analyze examples of the actual application of general criterion Eq. (1) to various physical and mechanical problems.

3 SPATIAL – TEMPORAL DISCRETENESS OF THE FRACTURE PROCESS

The incubation time fracture criterion, originally proposed (Petrov & Utkin 1989, Petrov 1991,Petrov & Morozov 1994) for predicting crack initiation under dynamic loading conditions, states that fracture will initiate at a point *x* at time *t* when

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{x-d}^{x}\sigma(x',t')\,dx'dt' \geq \sigma_c\ . \qquad (2)$$

Here, τ is the incubation time of the dynamic fracture process (or the fracture micro-structural time). It characterizes the response of the material to the applied dynamic loads; it is constant for a given material in the sense that it does not depend on the geometry of the test specimen, the way the load is applied, or the shape or amplitude of the load pulse. *d* is a characteristic size of the fracture process cell (zone) and is a constant for the given material and the chosen spatial scale. σ is the normal stress at the point which varies with time and σ_c is its critical value (i.e. the ultimate tensile strength evaluated under quasi-static conditions).

Assuming, as in the Irwin's small scale yielding approximation, that $d = \frac{2}{\pi}\frac{K_{Ic}^2}{\sigma_c^2}$. It can be shown that within the framework of linear elastic fracture mechanics (LEFM), the dynamic crack initiation criterion (2) for an existing mode I loaded crack is equivalent to $\frac{1}{\tau}\int_{t-\tau}^{t}K_I(t')\,dt' \leq K_{Ic}$. This follows the requirement that (2) is equivalent to the Irwin's criterion, $K_I(t) \geq K_{Ic}$, under quasi-static conditions ($t_*/\tau \to \infty$). This means that a certain size characterizing the fractured material appears in the dynamic fracture initiation criterion. This size is associated with the size of the failure cell on the current spatial scale – all ruptured cells sized less than *d* cannot be regarded as failure cells on the current scale level.

Thus, by the introduction of τ and *d* the temporal-spatial domain is discretized. Once the current working scale for a given material has been chosen, τ gives the time in which the energy accumulated in the cell of size *d* is enough to rupture it. We believe that a correct description of high loading rate effects requires the introduction of this temporal-spatial discreteness. The advantage of the incubation time approach is that one can remain within the framework of continuum linear elasticity and allow for the discreteness of the dynamic fracture process only inside the critical fracture condition.

As has been demonstrated previously (Petrov 1991, Petrov at al. 2004, Petrov & Sitnikova 2005, Bratov & Petrov 2007), the dynamic fracture criterion (2) successfully predicts fracture initiation in brittle solids. For slow loading rates when the times to fracture are much longer than τ, the criterion (2) is equivalent to the classic Irwin's criterion. For high loading rates when the times to fracture are comparable with τ, a variety of effects observed in dynamical experiments (Smith 1975, Ravi-Chandar & Knauss 1984, Kalthoff 1986, Dally & Barker 1988) has been explained qualitatively and quantitatively using Eq. (2) (Petrov 2004). The application of (2) for the description of real experiments or in the finite element analysis of dynamic fracture allows us to gain a better understanding of the nature of dynamic fracture and even to predict new effects typical for dynamical processes (Petrov & Sitnikova 2005, Bratov & Petrov 2007).

In order to utilize the incubation time approach for analysis of rock materials one needs to determine incubation process characteristics for particular rocks. Experiments on dynamic fracture of rock specimens were carried out at Research Center "Dynamics" of the St. Petersburg State University. Dynamic loading was created by magnetic field using experimental equipment developed by Krivosheev & Petrov (2004). An approach based on the incubation time concept was used to evaluate dynamic fracture toughness of the material.

Data presented in Table 1 was experimentally evaluated by Petrov et al (2005), the incubation time τ was found by analysis of threshold amplitudes of high-rate loads (Krivisheev & Petrov 2004), parameter *d* is calculated utilizing (2).

Table 1. Strength properties of some rock materials

N	Rock	σ_c , MPa	K_{Ic}, $\mathrm{MPa}\sqrt{\mathrm{m}}$	d , mm	τ , MS
1.	Limestone	12.40	1.31	7.11	15
2.	Gabbro-diabase	44.04	2.36	1.83	40
3.	Sandstone	31.18	1.19	0.93	54
4.	Granite	19.50	1.08	1.95	69
5.	Clay	1.63	0.12	3.45	75

Threshold (minimum fracturing) amplitudes for microsecond-range loads applied to faces of preexisting crack in plates made of different rocks were determined. Specimen sizes were "200x200x12 mm" for gabbro-diabase, "100x100x25 mm" for limestone, "300x300x10 mm" for granite, "120x120x30 mm" for clay, and "163x163x20 mm" for sandstone. Static mechanical properties for these materials were evaluated from data obtained in tests using standard material testing equipment.

Further we summarize some results connected with application of the incubation time approach to problems of dynamic fracture of rock materials. The incubation time based fracture criteria for intact media are discussed. A possibility to control external high-rate impact in order to optimize energy input for fracture of some of the rock materials is studied. It is can be shown (Smirnov et al. 2012) that optimal energy in order to initialize fracture in rock media does strongly depend on amplitude and duration of an impact causing this rupture.

4 SPALL FRACTURE CAUSED BY REFLECTED TENSILE STRESS WAVE

In case of “intact” (defect-free) media fracture criterion (2) can be rewritten:

$$\int_{t-\tau}^{t} \sigma(s)\, ds \le \sigma_c \tau \,. \tag{3}$$

Consider compressive triangularly symmetric shaped wave traveling along semi-infinite rod (Smirnov et al. 2012):

$$\sigma_-(x,t) = -P\left\{ \frac{ct+x}{ct_0}\; H(ct+x) - H(ct+x-ct_0)\; + \right.$$
$$\left. + \left(2 - \frac{ct+x}{ct_0}\right) H(ct+x-ct_0) - H(ct+x-2ct_0) \right\},$$

where P gives the pulse amplitude, $2t_0$ is the load duration, $H(t)$ is the Heaviside step function and c is the sound speed. The wave is reflecting from the stress-free end (x=0) of the rod and its sign is changed from compression to tension:

$$\sigma_+(x,t) = +P\left\{ \frac{ct-x}{ct_0}\; H(ct-x) - H(ct-x-ct_0)\; + \right.$$
$$\left. + \left(2 - \frac{ct-x}{ct_0}\right) H(ct-x-ct_0) - H(ct-x-2ct_0) \right\}.$$

The resulting stress in the rod is given by: $\sigma(x,t) = \sigma_-(x,t) + \sigma_+(x,t)$. Obviously, the maximum tensile stress firstly appears at the point $x_0 = ct_0/2$. Introducing dimensionless variables: $T = t/\tau$, $T_0 = t_0/\tau$, one can receive:

$$\sigma(T)\Big|_{x=x_0} = F(T) + G(T); \tag{4}$$

$$F(T) = -P\left\{ \left(\frac{1}{2} + \frac{T}{T_0}\right)\left[H\left(T + \frac{T_0}{2}\right) - H\left(T - \frac{T_0}{2}\right)\right] + \right.$$
$$\left. + \left(\frac{3}{2} - \frac{T}{T_0}\right)\left[H\left(T - \frac{T_0}{2}\right) - H\left(T - \frac{3T_0}{2}\right)\right]\right\};$$

$$G(T) = +P\left\{ \left(\frac{T}{T_0} - \frac{1}{2}\right)\left[H\left(T - \frac{T_0}{2}\right) - H\left(T - \frac{3T_0}{2}\right)\right] + \right.$$
$$\left. + \left(\frac{5}{2} - \frac{T}{T_0}\right)\left[H\left(T - \frac{3T_0}{2}\right) - H\left(T - \frac{5T_0}{2}\right)\right]\right\}.$$

The threshold (minimum) amplitude P_*, leading to fracture in the rod can be found utilizing fracture criterion (3) for any given duration t_0:

$$\max_T I(T) = \sigma_c \ , \quad I(T) = \int_{T-1}^{T} \sigma(s)\ ds \ . \qquad (5)$$

Obviously:

$$\max_T I(T) = I\left(\frac{3T_0}{2} + \frac{2}{3}\right) \quad \text{at} \quad T_0 \geq \frac{2}{3}$$

$$\max_T I(T) = I\left(T_0 + 1\right) \quad \text{at} \quad T_0 \leq \frac{2}{3}$$

i.e. time to fracture T_* can be calculated as:

$$T_* = \frac{3T_0}{2} + \frac{2}{3} \quad \text{at} \quad T_0 \geq \frac{2}{3}; \qquad T_* = T_0 + 1 \quad \text{at} \quad T_0 \leq \frac{2}{3} \ , \qquad (6)$$

and

$$\max_T I(T) = I\left(T_*\right) = P\left(1 - \frac{1}{3T_0}\right) \quad \text{at} \quad T_0 \geq \frac{2}{3};$$

$$\max_T I(T) = I\left(T_*\right) = \frac{3}{4} P T_0 \quad \text{at} \quad T_0 \leq \frac{2}{3} \ . \qquad (7)$$

Now, using (5) and (6) one can determine time to fracture T_* as a function of the threshold amplitude P_*

$$T_*(P_*) = \begin{cases} \dfrac{1}{2\left(1 - \sigma_c / P_*\right)} + \dfrac{2}{3} \ , & \text{at} \quad 1 \leq \dfrac{P_*}{\sigma_c} \leq 2 \ ; \\[2ex] \dfrac{4\sigma_c}{3P_*} + 1 \ , & \text{at} \quad \dfrac{P_*}{\sigma_c} \geq 2 \ . \end{cases} \qquad (8)$$

In dimensional variables:

$$t_* = \begin{cases} \dfrac{3}{2} t_0 + \dfrac{2}{3}\tau \ , & \text{at} \quad t_0 \geq \dfrac{2}{3}\tau \ ; \\[2ex] t_0 + \tau \ , & \text{at} \quad t_0 \leq \dfrac{2}{3}\tau \ . \end{cases} \qquad (9)$$

It is seen from the second expression in (9) that $t_* \to \tau$ as $t_0 \to 0$. Thus, the incubation time τ is the time to specimen fracture t_* while it is loaded by *threshold* pulse of infinitesimal duration (i.e. by pulse having the Dirac delta-function form). At threshold loads (with amplitudes equal to P_*) the time to fracture cannot be shorter than τ, – a certain period of time (incubation time) is needed for the material "to prepare" fracture. The time to fracture can be less than the incubation time only in case of over threshold loads, i.e. at overloaded impacts.

Analysis of temporal strength dependence gives a possibility to draw important conclusions about interrelation and evidence variety of quasistatic and dynamic spall fracture mechanisms. The resultant diagram of temporal strength dependence (Figure 2) is the main characteristic of spall strength. One can calculate (threshold) momentum corresponding to the threshold loads leading to spall fracture. It can be calculated as $U_*(t_0) = P_* \cdot t_0$ for the studied time shape of the load (isosceles triangle). The threshold amplitude P_* can be found from (4) and (5):

$$P_*(t_0) = \begin{cases} \dfrac{\sigma_c}{1-\dfrac{\tau}{3t_0}}, & \text{at} \quad t_0 \geq \dfrac{2}{3}\tau; \\ \dfrac{4\sigma_c}{3t_0}\tau, & \text{at} \quad t_0 \leq \dfrac{2}{3}\tau. \end{cases}$$

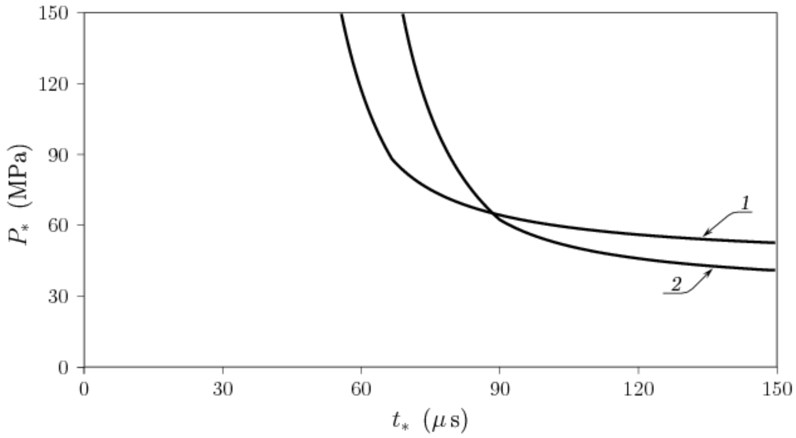

Figure 2. Temporal spall strength dependence of rocks: *1* – gabbro-diabase, *2* – sandstone.

It is evident that the fracture threshold is essentially determined by both the dynamic fracture parameter τ and the static strength of material. The static branch (long loads, low threshold amplitudes) is fully controlled by the static material strength σ_c, while the dynamic branch (short loads, higher threshold amplitudes) is mainly controlled by the fracture incubation time τ. As can be seen from Figure 2, even though gabbro-diabase has larger quasistatic tensile strength, in conditions of high-rate loading it appears to be easier to fracture as comparing to sandstone.

5 DYNAMIC TENSILE FRACTURE IN SPLIT CYLINDER TESTS

The splitting method was used for determining of dynamic tensile strength of fibre-reinforced concrete CARDIFRC (Bragov et al. 2012). This method was originally proposed for determining the quasi-static tensile strength of brittle materials. However, numerous authors (see. ex: Gama et al. 2004, Rodriguez et al. 1994, Bragov et al. 2008) carried out experimental and numerical analysis of fracture of materials under a splitting force and concluded that the splitting tests can be also be employed to determine the dynamic tensile strength of brittle media provided that the elastic behaviour and the equilibrium state are ensured, and the failure is produced in a predictable manner. The difference of the incident and reflected pulses is practically equal to the transmitted pulse so that the forces on the specimen are in equilibrium. Figure 3 shows the photo of a partially damaged specimen with the split along the diametrical plane. A layer of graphite grease was applied to the contact areas between the measuring bars and the specimen to reduce the influence of friction.

Figure 3. Photo of a partially damaged specimen in split test (Bragov et al. 2012).

The dynamic split cylinder test was conducted to determine the threshold value of the splitting stress and to investigate the influence of stress rate on the indirect tensile strength. For this purpose three test regimes were employed: regime 1 – preservation of the integrity of the sample, regime 2 – partial damage of the sample (Figure 3), regime 3 – split fracture. The typical time stress profiles are shown in Figure 4. These profiles were obtained after processing the electric pulses according to relation (2).

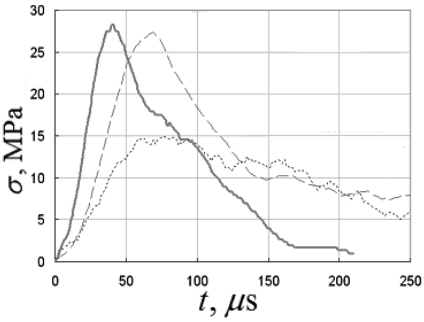

Figure 4. The time variation of stress σ used in the dynamic split tests of CARDIFRC. The solid curve represents fracture of the sample, the dashed line - partial fracture of the sample, and the dotted line – no visible damage of the sample (Bragov et al. 2012).

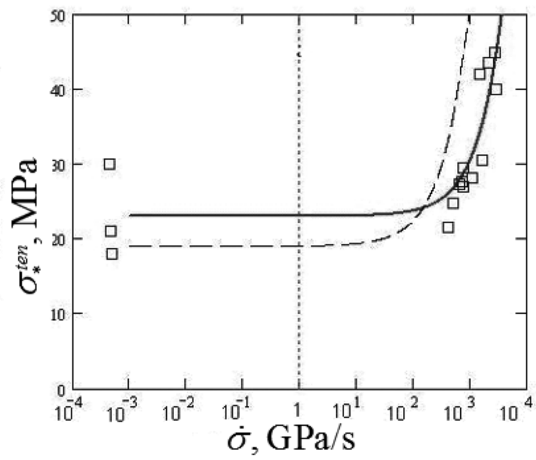

Figure 5. The dynamic split strength of CARDIFRC; experimental values (open squares), predictions of Eq. 10 (solid line) ($\sigma_c^{tensile}$ = 23 MPa and τ = 15 μs). The dashed line shows the predicted strength of granite for comparison ($\sigma_c^{tensile}$ = 19 MPa and τ = 70 μs - Petrov et al. 2005)

The dynamic splitting tensile strength was predicted based on the criterion of the incubation time, which in this case takes the following form:

$$\frac{1}{\tau}\int_{t-\tau}^{t} \sigma\ (t')\ dt' \le \sigma_c^{tensile}, \qquad (10)$$

where $\sigma(t)$ is the temporal dependence of the tensile stress in the centre of the sample; $\sigma_c^{tensile}$ is the static indirect tensile strength, and τ is the incubation time of fracture of the material in tension.

Figure 5 summarizes the results of quasi-static and dynamic testing of CARDIFRC in splitting. The continuous curve corresponds to the calculation by criterion (Eq. 10) for the following values of the parameters: $\sigma_c^{tensile}$ = 23 MPa and τ = 15 μs. The dashed curve shows for comparison the calculation by (Eq. 10) the rate dependence of granite strength with the following parameters: $\sigma_c^{tensile}$ = 19 MPa and τ = 70 μs. As is clear from Figure 5, CARDIFRC has a lower dynamic split strength for a higher static split strength compared to granite. Thus, the dynamic split strength of CARDIFRC in the studied range of stress rate ~$500\cdot10^3$ MPa/s $\le \dot{\sigma} \le$ ~$3000\cdot10^3$ MPa/s increases, and can be effectively predicted by the incubation time criterion.

6 CONCLUSIONS

The central problem of testing the dynamic strength properties of brittle materials like rock and concrete can be associated with measurements of the incubation time parameter. Studies of threshold characteristics (pulse amplitudes, time to fracture, etc.) of fracture processes provide an effective opportunity to examine the incubation stage of the fracture process and to evaluate a set of fixed material parameters for the structural-time criterion. Different experiments (i.e. spall fracture, split cylinder tests) can be interpreted within the framework of a single theory using the structural-temporal (incubation time based) approach. It is shown that the time dependence of the dynamic tension and split tensile strengths can be predicted by the incubation time criterion. Obviously any of these experimental schemes can be used for independent dynamic testing of materials. It is shown that in spite of the fact that static strength of one material is smaller than that of another one, its dynamic strength measured in terms of incubation time can be essentially higher.

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